IX. MICROWAVE ELECTRONICS*

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RESEARCH OBJECTIVES

We are continuing the study of high-density electron beams. Two ways of producing such beams are by means of the magnetron injection gun and the hollow cathode. The theory of magnetron injection guns is being studied, and a design of such a gun has been completed. Radiofrequency interaction experiments for $\omega \sim \omega_p$ will be made with the use of such a hollow beam. The study of the theory of waves in high-density electron-beam waveguides will also continue.

L. D. Smullin, A. Bers

A. HIGH-PERVEANCE HOLLOW ELECTRON-BEAM STUDY

The design of a new magnetron injection gun has been started. This gun employs a conical cathode immersed in a uniform magnetic field, as shown in Fig. IX-1. The

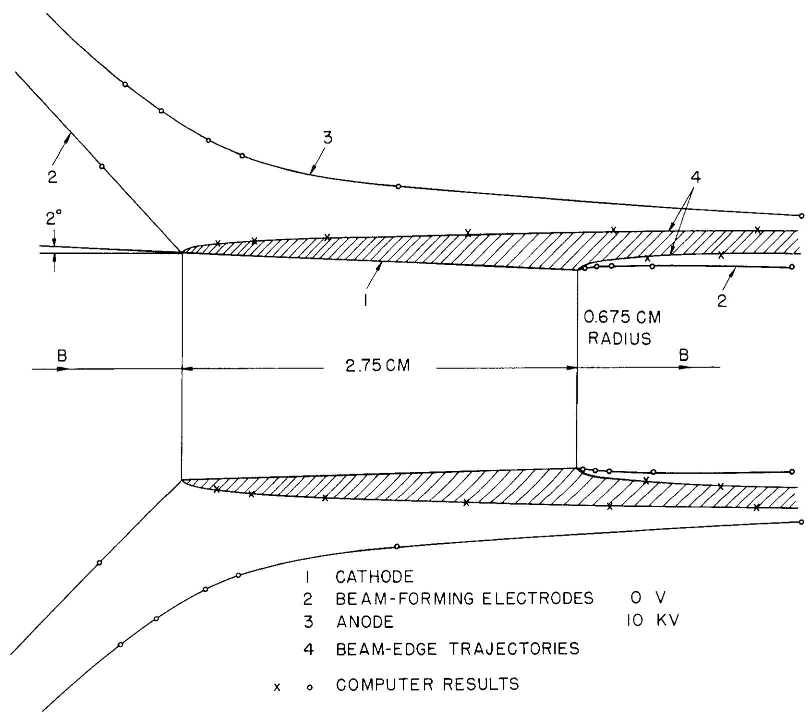

Fig. IX-1. Two-dimensional magnetron injection gun computer solution.

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shape of the beam-forming electrodes has been calculated on a computer by using a planar model of the gun and correcting for the differences between the planar and the axisymmetrical case. The design data are:

Gun	
Perveance	$K = 10\ \mu aV^{-3/2}$
Voltage	V = 10 KV
Cone half-angle	$\theta = 2°$
Cathode current density	$J_c = 0.8$ amp cm^{-2}
Magnetic field	B = 1000 gauss
Beam in Drift Tube	
Diameter	2b = 1.6 cm
Thickness	15 per cent of beam radius
Clearance to drift tube	20 per cent of beam radius
Beam current density	$J_b = 18$ amp cm^{-2}.

The construction of a demountable beam-testing system has been completed. The hollow beam from this gun will be tested for its dc properties and signal-interaction characteristics.

A. Bers, A. Poeltinger

B. FAST WAVES IN HIGH-DENSITY ELECTRON-BEAM WAVEGUIDES

In previous reports[1,2] the problem of the slow waves that can exist on an electron beam which is of arbitrary density and focused by an infinite magnetic field has been considered. In the present report we would like to establish some general characteristics of the fast waves in such a structure. In particular, we wish to show that the cutoff frequency of these fast waves is always greater than the cutoff frequency associated with the empty waveguide, even for arbitary beam densities.

To establish these results, we shall consider, first, a stationary electron gas in a circular waveguide neutralized by very heavy ions. The ω-β relation for the electron-beam case can be derived from the stationary case by a simple transformation. This transformation is

$$\omega = \frac{\omega' + \beta' v_o}{\sqrt{1 - v_o^2/c^2}} \tag{1}$$

$$\beta = \frac{\beta' + \omega' v_o/c^2}{\sqrt{1 - v_o^2/c^2}} \tag{2}$$

where (ω', β') refers to the stationary electron gas, and (ω, β) refers to an electron beam moving with time-average velocity v_o in the +z direction. It is easily shown that the quantity

$$I = k^2 - \beta^2, \qquad k = \frac{\omega}{c} \tag{3}$$

is an invariant of the transformation; that is, any hyperbola defined by a particular value of I in Eq. 3 will transform into the same hyperbola in the new reference frame.

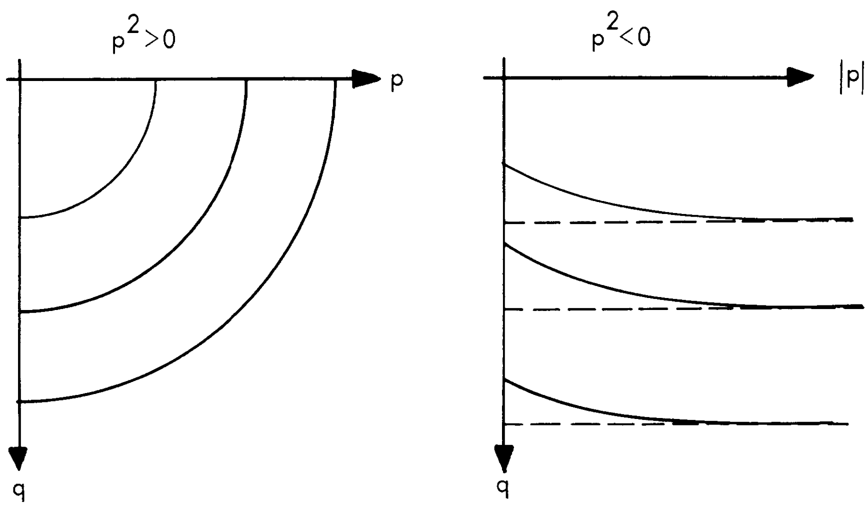

Fig. IX-2. Sketch of the determinantal equation from the boundary conditions. (Circular cylindrical geometry.)

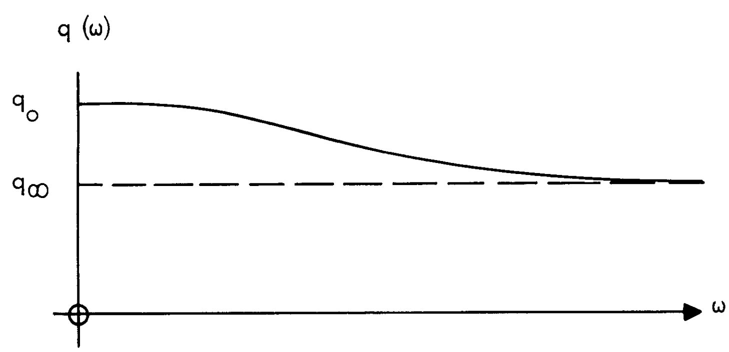

Fig. IX-3. Sketch of transverse wave number q as a function of frequency.

For the electron gas at rest, and a circular waveguide cross section the fields vary as Bessel functions with the argument (pr) inside the beam, and with (qr) outside the beam, with

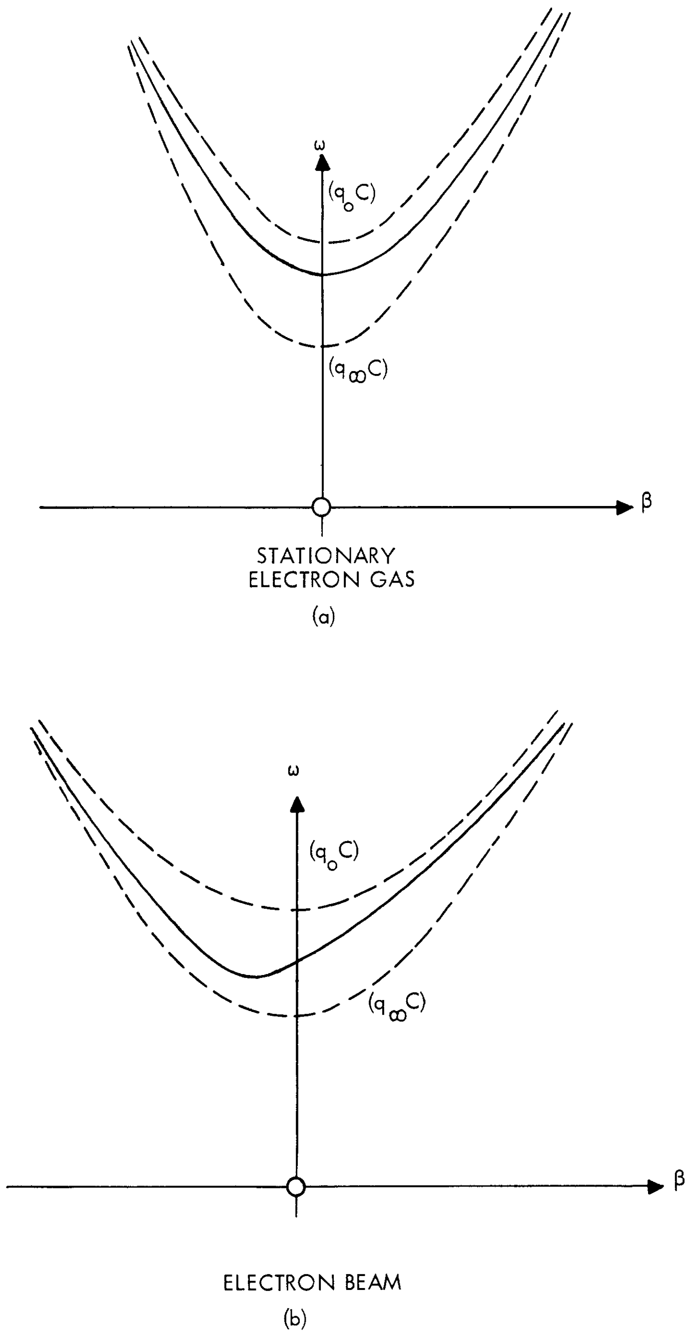

Fig. IX-4. Dispersion characteristics for fast waves; the slow waves are not shown.

$$p^2 = q^2\left(1-\omega_p^2/\omega^2\right) \tag{4}$$

$$q^2 = k^2 - \beta^2. \tag{5}$$

Another relation between p and q is obtained from the boundary conditions. The simultaneous solution of (4) and the boundary-condition equation will determine $q(\omega)$; use of this in (5) will yield the relation between ω and β.

If we are interested in fast waves, then $\beta^2 < k^2$ and $q^2 > 0$. From (4), we see that

$$p^2 > 0 \qquad \text{for } \omega > \omega_p$$

$$p^2 < 0 \qquad \text{for } \omega < \omega_p.$$

The relation between p and q is qualitatively sketched in Fig. IX-2 for a solid beam.[3] If we plot the straight-line relation between p and q as given by (4), we can see that $q(\omega)$ for a particular mode will be a monotonically decreasing function of ω, as sketched in Fig. IX-3. We notice that as $\omega \to \infty$, $p \to q$; and hence $q \to q_\infty$, where q_∞ must be the radial wave number of the waveguide with no beam present. Conversely, as $\omega \to 0$, $p^2 < 0$ and $|p| \to \infty$; this means that the q that results corresponds physically to the radial wave number of the higher order TM modes in a system composed of the waveguide and a perfectly conducting tube which replaces the beam (no fields inside the beam).

The form of the ω-β plot for the stationary electron gas is sketched in Fig. IX-4a. The curve is necessarily bounded by the hyperbolas shown in the figure. Since every point on the ω-β curve lies on a hyperbola bounded by the hyperbolas shown, the ω-β plot for the electron beam (shown in Fig. IX-4b) will also be bounded by these hyperbolas. The lowest possible frequency is no longer at $\beta = 0$; however, this lowest frequency is bounded by $(q_o c)$ and $(q_\infty c)$. This establishes the result that we desired.

To summarize: (a) Fast waves on the electron-beam system will definitely not exist as long as the frequency ω is less than the cutoff frequency of the empty waveguide. (b) Fast waves definitely will exist if ω is greater than the lowest cutoff frequency of the higher order TM mode in a system composed of the waveguide and a perfect conductor that replaces the beam.

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References

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2. C. G. Alexander and A. Bers, Slow backward waves in electron-beam waveguides, Quarterly Progress Report No. 61, Research Laboratory of Electronics, M. I. T., April 15, 1961, pp. 58-62.

3. C. G. Alexander, S. M. Thesis, Department of Electrical Engineering, M. I. T., September 1960.