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ESTIMATION OF INERTIAL PARAMETERS OF RIGID BODY LINKS OF MANIPULATORS

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Abstract. A method of estimating the mass, the location of center of mass, and the moments of inertia of each rigid body link of a robot during general manipulator movement is presented. The algorithm is derived from the Newton-Euler equations, and uses measurements of the joint torques as well as the measurement and calculation of the kinematics of the manipulator while it is moving. The identification equations are linear in the desired unknown parameters, and a modified least squares algorithm is used to obtain estimates of these parameters. Some of the parameters, however, are not identifiable due to the restricted motion of proximal links and the lack of full force/torque sensing. The algorithm was implemented on the MIT Serial Link Direct Drive Arm. A good match was obtained between joint torques predicted from the estimated parameters and the joint torques computed from motor currents.

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1 Introduction

This paper presents a method of estimating all of the inertial parameters, the mass, the center of mass, and the moments of inertia of each rigid body link of a robot manipulator using joint torque sensing. Determining these parameters from measurements or computer models is generally difficult and involves some approximations to handle the complex shapes of the arm components. Typically, even the manufacturers of manipulators do not know accurate values of these parameters.

The degree of uncertainty in inertial parameters is an important factor in judging the robustness of model-based control strategies. A common objection to the computed torque methods, which involve full dynamics computation (e.g., Luh, Walker, and Paul, 1980), is their sensitivity to modelling errors, and a variety of alternative robust controllers have been suggested (Samson, 1983; Slotine, 1984; Spong, Thorp, and Kleinwaks; 1984, Gilbert and Ha, 1984). Typically these robust controllers express modelling errors as a differential inertia matrix and coriolis and gravity vectors, but in so doing, no rational basis is provided for the source of errors or the bounds on errors. The error matrices and vectors combine kinematic and dynamic parameter errors, but kinematic calibration is sufficiently developed so that very little error can be expected in the kinematic parameters (Whitney, Lozinski, and Rourke , 1984).

One aim of this work is to place similar bounds on inertial parameter errors by explicitly identifying the inertial parameters of each link that go into the making of the inertia matrix and coriolis and gravity vectors. Our work in load identification (Atkeson, An, and Hollerbach, 1985) suggests, for example, that mass can be accurately identified to within 1%. Therefore, an assumption of 50% error in link mass in verifying a robust control formulation (Spong, Thorp, and Kleinwaks, 1984) is an unreasonable basis for argument. Slotine (1984) suggests that errors of only a few percent in inertial parameters make his robust controller superior to the computed torque method, but it may well be that these parameters can be identified more accurately than his assumptions.

As an alternative approach we propose estimating the inertial parameters on the basis of direct dynamic measurements. The same algorithms used to identify load inertial parameters (Atkeson, An, and Hollerbach, 1985) can be modified to find link inertial parameters of a robot arm made up of rigid parts. The Newton-Euler dynamic equations are used to express the measured forces and torques at each joint in terms of the product of the measured movements of the rigid body links and the unknown link inertial parameters. These equations are linear in the inertial parameters. However, unlike load estimation, the only sensing is one component of joint torque, inferred from motor current. Coupled with restricted movement near the base, it is, therefore, not possible to find all the inertial parameters of the

proximal links. As will be seen, these missing parameters have no effect on the control of the arm.

In this paper, manipulators with only revolute joints are discussed since handling prismatic joints requires only trivial modifications to the algorithm. The proposed algorithm was verified by implementation on the MIT Serial Link Direct Drive Arm.

1.1 Previous Work

Mayeda, Osuka, and Kangawa (1984) required three sets of special test motions to estimate the coefficients of a closed-form Lagrangian dynamics formulation. The 10 inertial parameters of each link are lumped into these numerous coefficients, which are redundant and susceptible to numerical problems in estimation. On the other hand, every coefficient is identifiable since these coefficients represent the actual degrees of freedom of the robot. By sensing torque from only one joint at a time, their algorithm is more susceptible to noise from transmission of dynamic effects of distant links to the proximal measuring joints. For efficient dynamics computation, the recursive dynamics algorithms require the link parameters explicitly. Though recoverable from the Lagrangian coefficients, it is better to estimate the inertial parameters directly. Though this algorithm was implemented on a PUMA robot, it is difficult to interpret the results because of dominance of the dynamics by the rotor inertia and friction.

Mukerjee (1985) directly applied his load identification method to link identification, by requiring full force-torque sensing at each joint. Instrumenting each robot link with full force-torque sensing seems impractical, and is actually unnecessary given joint torque sensing about the rotation axis. Partially as a result, he does not address the issue of unidentifiability of some inertial parameters. Also, he did not verify his algorithm by simulation or by implementation.

Olsen and Bekey (1985) presented a link identification procedure using joint torque sensing and special test motions with single joints. The unidentifiability of certain inertial parameters was not resolved, and the least squares estimation procedure written as a generalized inverse would fail because of linear dependence of some of the inertial parameters. Again, their procedure was not tested by simulation or by actual implementation on a robot arm.

Neuman and Khosia (1985) developed a hybrid estimation procedure combining a Newton-Euler and a Lagrange-Euler formulation of dynamics. Simulation results for a three degree-of-freedom cylindrical robot were presented, and the unidentifiability of certain inertial components was addressed. For some reason they state link mass must be known for a linear estimation procedure, but such a restriction does not exist with our method. Though planning to work with the CMU DDArm II, they have not yet presented experimental results.

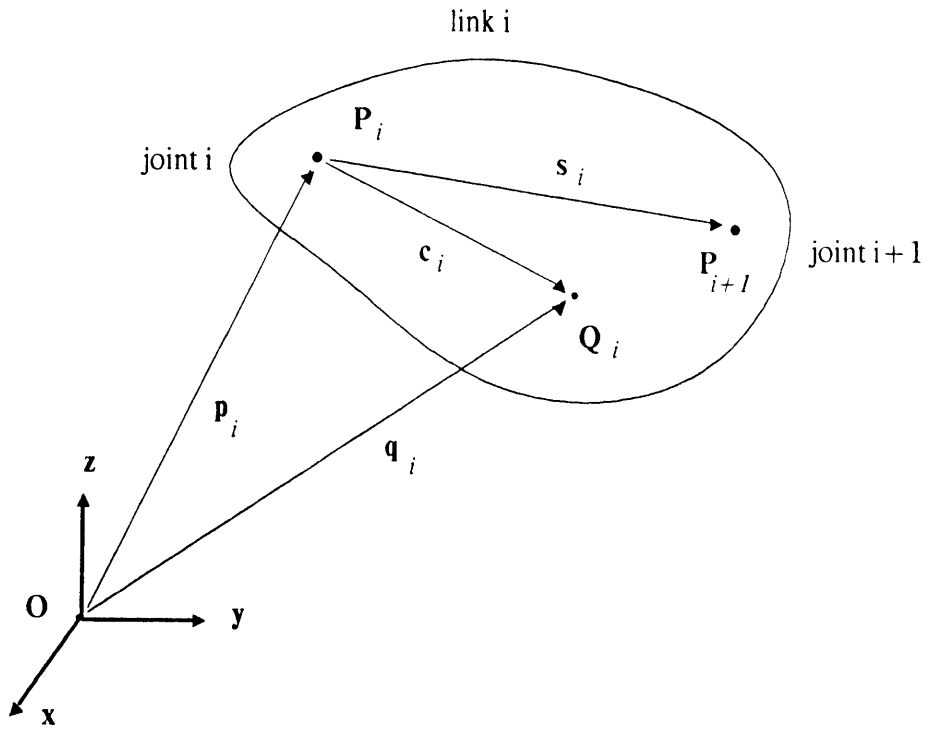

Figure 1: Coordinate origins and location vectors for link identification.

2 ESTIMATION PROCEDURE

2.1 Formulation of Newton-Euler Equations

In our work in load identification (Atkeson, An, and Hollerbach, 1985), the Newton-Euler equations for a rigid body load were formulated to be linear in the unknown inertial parameters. Then simple linear least squares method was used to estimate those parameters. By treating each link of a manipulator as a load, this formulation can be extended to the link estimation problem. The differences in the equations are that only one component of force or torque is sensed and that the forces and torques from distal links are summed and transmitted to the proximal joints.

Consider a manipulator with n joints (Figure 1). Each link i has its own local coordinate system $\mathbf{P_i}$ fixed in the link with its origin at joint i. The joint force and torque due to the movement of its own link can be expressed by simply treating the link as a load and applying the previously developed equations for load identification (Atkeson, An, and Hollerbach, 1985):

$$\begin{bmatrix} \mathbf{f}_{ii} \\ \mathbf{n}_{ii} \end{bmatrix} = \begin{bmatrix} \ddot{\mathbf{p}}_i - \mathbf{g} & [\dot{\omega}_i\times] + [\omega_i\times][\omega_i\times] & \mathbf{0} \\ \mathbf{0} & [(\mathbf{g} - \ddot{\mathbf{p}}_i)\times] & [\bullet\dot{\omega}_i] + [\omega_i\times][\bullet\omega_i] \end{bmatrix} \begin{bmatrix} m_i \\ m_i c_{x_i} \\ m_i c_{y_i} \\ m_i c_{z_i} \\ I_{xx_i} \\ I_{xy_i} \\ I_{xz_i} \\ I_{yy_i} \\ I_{yz_i} \\ I_{zz_i} \end{bmatrix}$$

or more compactly,

$$\mathbf{w}_{ii} = \boldsymbol{A}_i \boldsymbol{\phi}_i \tag{1}$$

where $\mathbf{w}_{ij}$ is the wrench (vector of forces and torques) at joint i due to movement of link j alone. $\boldsymbol{A}_i$ is the kinematic matrix that describes the motion of link i and $\boldsymbol{\phi}_i$ is the vector of unknown link inertial parameters. All of the quantities are expressed in the local joint i coordinate system. The formulation of the above Newton-Euler equations were already presented in the load identification paper (Atkeson, An, and Hollerbach), and are summarized in the Appendix of this paper for a reference.

The total wrench $\mathbf{w}_i$ at joint i is the sum of the wrenches $\mathbf{w}_{ij}$ for all links j distal to joint i:

$$\mathbf{w}_i = \sum_{j=i}^{N} \mathbf{w}_{ij} \tag{2}$$

Each wrench $\mathbf{w}_{ij}$ at joint i is determined by transmitting the distal wrench $\mathbf{w}_{jj}$ across intermediate joints. This is a function of the geometry of the linkage only. The forces and torques at neighboring joints are related by

$$\begin{bmatrix} \mathbf{f}_{i,i+1} \\ \mathbf{n}_{i,i+1} \end{bmatrix} = \begin{bmatrix} \mathbf{R}_i & \mathbf{0} \\ [\mathbf{s}_i\times] \cdot \mathbf{R}_i & \mathbf{R}_i \end{bmatrix} \begin{bmatrix} \mathbf{f}_{i+1,i+1} \\ \mathbf{n}_{i+1,i+1} \end{bmatrix} \tag{3}$$

or more compactly

$$\mathbf{w}_{i,i+1} = \mathbf{T}_i \, \mathbf{w}_{i+1,i+1} \tag{4}$$

where

$\mathbf{R}_i$= the rotation matrix rotating the link $i+1$ coordinate system to the link i coordinate system,

$\mathbf{s}_i$= a vector from the origin of the link i coordinate system to the link $i+1$ coordinate system, and

$\mathbf{T}_i$= a wrench transmission matrix.

To obtain the forces and torques at the i^{th} joint due to the movements of the j^{th} link, these matrices can be cascaded:

$$\begin{aligned} \mathbf{w}_{ij} &= \mathbf{T}_i \mathbf{T}_{i+1} \cdots \mathbf{T}_j \mathbf{w}_{jj} \\ &= \mathbf{U}_{ij} \phi_j \end{aligned} \tag{5}$$

where $\mathbf{U}_{ij} = \mathbf{T}_i \mathbf{T}_{i+1} \cdots \mathbf{T}_j \boldsymbol{A}_i$ and $\mathbf{U}_{ii} = \boldsymbol{A}_i$. A simple matrix expression for a serial kinematic chain (in this case a six joint arm) can be derived from (2) and (5):

$$\begin{bmatrix} \mathbf{w}_1 \\ \mathbf{w}_2 \\ \mathbf{w}_3 \\ \mathbf{w}_4 \\ \mathbf{w}_5 \\ \mathbf{w}_6 \end{bmatrix} = \begin{bmatrix} \mathbf{U}_{11} & \mathbf{U}_{12} & \mathbf{U}_{13} & \mathbf{U}_{14} & \mathbf{U}_{15} & \mathbf{U}_{16} \\ \mathbf{0} & \mathbf{U}_{22} & \mathbf{U}_{23} & \mathbf{U}_{24} & \mathbf{U}_{25} & \mathbf{U}_{26} \\ \mathbf{0} & \mathbf{0} & \mathbf{U}_{33} & \mathbf{U}_{34} & \mathbf{U}_{35} & \mathbf{U}_{36} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{U}_{44} & \mathbf{U}_{45} & \mathbf{U}_{46} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{U}_{55} & \mathbf{U}_{56} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{U}_{66} \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \\ \phi_6 \end{bmatrix} \tag{6}$$

This equation is linear in the unknown parameters, but the left side is composed of a full force-torque vector at each joint. Since only the torque about the joint axis can usually be measured, each joint wrench must be projected onto the joint rotation axis (typically $[0, 0, 1]$ in internal coordinates), reducing (6) to

$$\tau = \boldsymbol{K}\psi \tag{7}$$

where $\tau_i = [0, 0, 0, 0, 0, 1] \cdot \mathbf{w}_i$ is the joint torque of the i^{th} link, $\psi = [\phi_1, \phi_2, \phi_3, \phi_4, \phi_5, \phi_6]^T$, and $\boldsymbol{K}_{ij} = [0, 0, 0, 0, 0, 1] \cdot \mathbf{U}_{ij}$ when the corresponding entry in (6) is nonzero. For an n-link manipulator, τ is a $n \times 1$ vector, ψ is a $10n \times 1$ vector, and $\boldsymbol{K}$ is a $n \times 10n$ matrix.

2.2 Estimating the Link Parameters

Equation (7) represents the dynamics of the manipulator for one sample point. For extra data, (7) is augmented as:

$$\mathbf{K} = \begin{bmatrix} \boldsymbol{K}(1) \\ \cdot \\ \cdot \\ \boldsymbol{K}(N) \end{bmatrix} \quad \tau = \begin{bmatrix} \tau(1) \\ \cdot \\ \cdot \\ \tau(N) \end{bmatrix} \quad N = \text{number of data points}$$

Unfortunately, one cannot apply simple least squares estimate:

$$\psi_{estimate} = (\mathbf{K}^T \mathbf{K})^{-1} \mathbf{K}^T \tau \tag{8}$$

because $\mathbf{K}^T\mathbf{K}$ is not invertible due to loss of rank from restricted degrees of freedom at the proximal links and the lack of full force-torque sensing.

There are several ways to resolve this problem. One way to resolve this problem is to use the pseudo inverse to get a solution $\hat{\psi}$ to $\tau = \mathbf{K}\psi$. But since $\mathbf{K}$ is a potentially large $nN \times 10n$ matrix, the pseudo-inverse is computationally inefficient. Another simple method similar to the pseudo inverse is to use ridge regression (Marquardt and Snee, 1975). Ridge regression makes $\mathbf{K}^T\mathbf{K}$ invertible by adding a small number to the diagonal elements:

$$\hat{\psi} = (\mathbf{K}^T\mathbf{K} + d\mathbf{I}_{10n})^{-1}\mathbf{K}^T\tau \tag{9}$$

The estimates are nearly optimal if $d << \lambda_{min}(\mathbf{K}^T\mathbf{K})$, where λ_{min} is the smallest non-zero eigenvalue of $\mathbf{K}^T\mathbf{K}$. Other methods of solution, fundamentally different from the above two, are presented in the discussion section.

3 Experimental Results

Link estimation was implemented on the MIT Serial Link Direct Drive Arm (Figure 2), a three link serial manipulator with no transmission mechanism between the motors and the links. The ideal rigid body dynamics is a good model for this arm, uncomplicated by joint friction or backlash typical of other manipulators. Hence the fidelity of this manipulator's dynamic model suits estimation well. The coordinate system for this arm is defined in Figure 3. A set of inertial parameters is available for the arm (Table 1), determined by modeling with a CAD/CAM database (Lee, 1983). These values can serve as a point of comparison for our method, but they may not be accurate because of modeling errors.

The motors are rated at 660 Nm peak torque for joint 1 and 230 Nm for joints 2 and 3 (Asada and Youcef-Toumi, 1984). Joint 1 is presently capable of an angular acceleration of 1150 deg/sec^2, joints 2 and 3 in excess of 6000 deg/sec^2. In comparision, joint 1 of the PUMA 600 has a peak acceleration of 130 deg/sec^2 and joint 4 at the wrist 260 deg/sec^2. Joint position is measured by a resolver and joint velocity by a tachometer. The tachometer output is of high quality and leads to good acceleration estimates after differentiation. The accuracy of the acceleration estimates plus high angular accelerations greatly improves inertia estimation.

The joint torques are computed by measuring the currents of the 3 phase windings of each motor (Asada, Youcef-Toumi, and Lim, 1984). For the three phase brushless permanent magnet motors of the direct drive arm, the output torque is related to the currents in the windings by:

$$\tau = K_T(I_a \sin\theta + I_b \sin(\theta + 120) + I_c \sin(\theta + 240)) \tag{10}$$

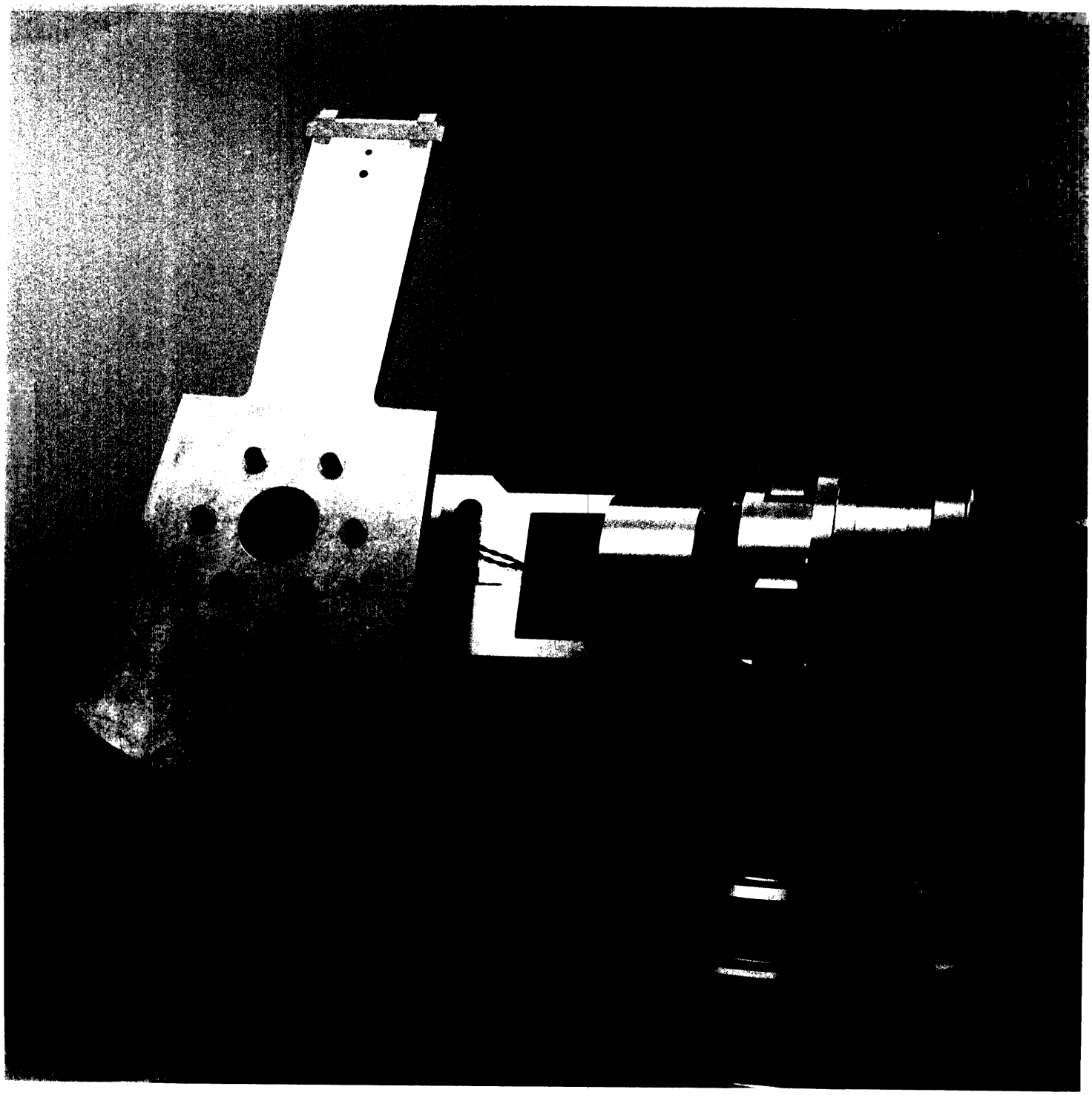

Figure 2: The MIT Serial Link Direct Drive Arm

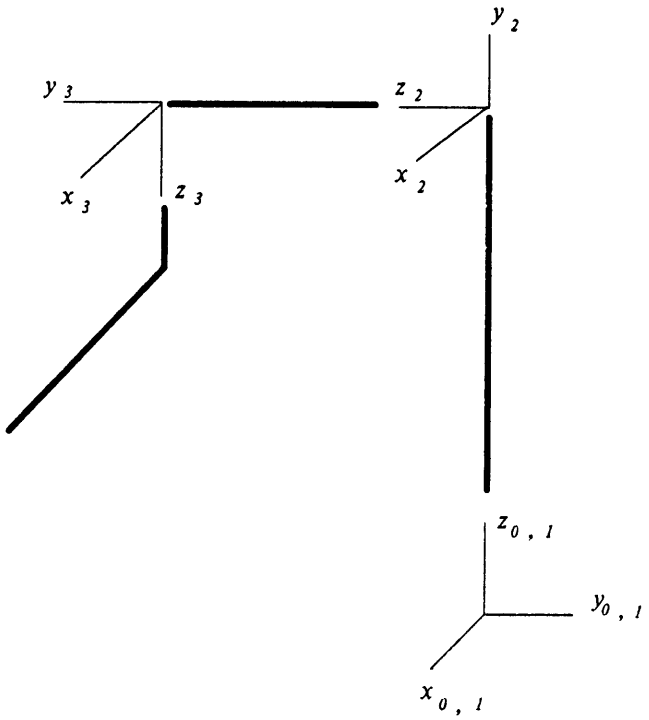

Figure 3: The link coordinate system.

The torque constant K_T for each motor is calibrated statically by measuring the force produced by the motor torque at the end of a known lever arm. The force is measured using a BarryWright Company Astek FS6-10A-200 6-axis force/torque sensor. Asada, Youcef-Toumi, and Lim have found that for a motor similar to the motors of our manipulator, the torque versus current relationship was nonlinear, especially for small magnitudes of torques, and also varied as a function of the rotor position. However, for the results presented in this paper, the nonlinear effects were ignored since substantial portions of the movements in the experiments required large magnitudes of torques. Since the least squares algorithm minimizes the square of the error, torque errors for torques of small magnitudes do not affect the estimates very much.

For the estimation results presented, 600 data points were sampled while the manipulator was executing 3 sets of fifth order polynomial trajectories in joint space. The specifications of the trajectories were:

1. (330, 289.1, 230) to (80, 39.1, -10) degrees in 1.3s,
2. (330, 269.1, -30) to (80, 19.1, 220) degrees in 1.3s,
3. (80, 269.1, -30) to (330, 19.1, 220) degrees in 1.3s,

Since $\mathbf{K}^T\mathbf{K}$ in (9) is singular, estimates for the 30 unknowns are computed by adding a small number ($d = 10.0 << \lambda_{min}(\mathbf{K}^T\mathbf{K}) = 3395.0$) to the diagonal elements of

Parameters	Link 1	Link 2	Link 3
$m(Kg)$	67.13	53.01	19.67
$mc_x(Kg \cdot m)$	0.0	0.0	0.3108
mc_y	2.432	3.4081	0.0
mc_z	35.8257	16.6505	0.3268
$I_{xx}(Kg \cdot m^2)$	23.1568	7.9088	0.1825
I_{xy}	0.0	0.0	0.0
I_{xz}	-0.3145	0.0	-0.0166
I_{yy}	20.4472	7.6766	0.4560
I_{yz}	-1.2948	-1.5036	0.0
I_{zz}	0.7418	0.6807	0.3900

Table 1: CAD-modeled inertial parameters.

$\mathbf{K}^T\mathbf{K}$.

Typical results, obtained using ridge regression method, are shown in Table 2. Parameters that cannot be identified because of constrained motion near the base are denoted by 0.0*. The first nine parameters of the first link are not identifiable because this link has only one degree of freedom about its **z** axis. These nine parameters do not matter at all for the movement of the manipulator and thus can be arbitrarily set to 0.0.

Other parameters marked by (†) can only be identified in linear combinations, indicated explicitly in Table 3. The ridge regression automatically resolves the linear combinations in a least squares sense. It can be seen that the estimated sums roughly match the corresponding sums inferred from the CAD-modeled parameters, but the sizeable discrepancy indicates that one parameter set may be more accurate than the other.

To verify the accuracy of the estimated and the modeled parameters, the measured joint torques are compared to the torques computed from the above two sets of parameters using the measured joint kinematic data. As shown in Figure 4, the estimated torques match the measured torques very closely. The torques computed from the CAD/CAM modeled parameters do not match the measured torques as closely. This comparison verifies qualitatively that for control purposes the estimated parameters are in fact more accurate than the modeled parameters.

Parameters	Link 1	Link 2	Link 3
$m(Kg)$	0.0*	0.0*	1.8920†
$mc_x(Kg \cdot m)$	0.0*	-0.1591	0.4676
mc_y	0.0*	0.6776†	0.0315
mc_z	0.0*	0.0*	-1.0087†
$I_{xx}(Kg \cdot m^2)$	0.0*	4.1562†	1.5276†
I_{xy}	0.0*	0.3894	-0.0256
I_{xz}	0.0*	0.0118	0.0143
I_{yy}	0.0*	5.2129†	1.8967†
I_{yz}	0.0*	-0.6050†	-0.0160
I_{zz}	9.33598†	-0.8194†	0.3568

Table 2: Estimated inertial parameters.

Linear Combinations	Estimated	CAD-Modeled
$m_3c_{z_3}l_2 + I_{yz_2}$	-1.0589	-1.3565
$I_{xx_3} - I_{yy_3}$	-0.3691	-0.2702
$I_{zz_2} + I_{xx_3}$	0.7082	0.8632
$I_{zz_1} + I_{xx_2} + I_{xx_3} + m_3l_2^2$	15.7029	12.8169
$I_{xx_2} + I_{xx_3} - I_{yy_2}$	0.4709	0.4147
$m_3c_{z_3} - m_2c_{y_2}$	-1.6863	-3.0814

Table 3: Parameters in linear combinations ($l_2 = 0.45m$.)

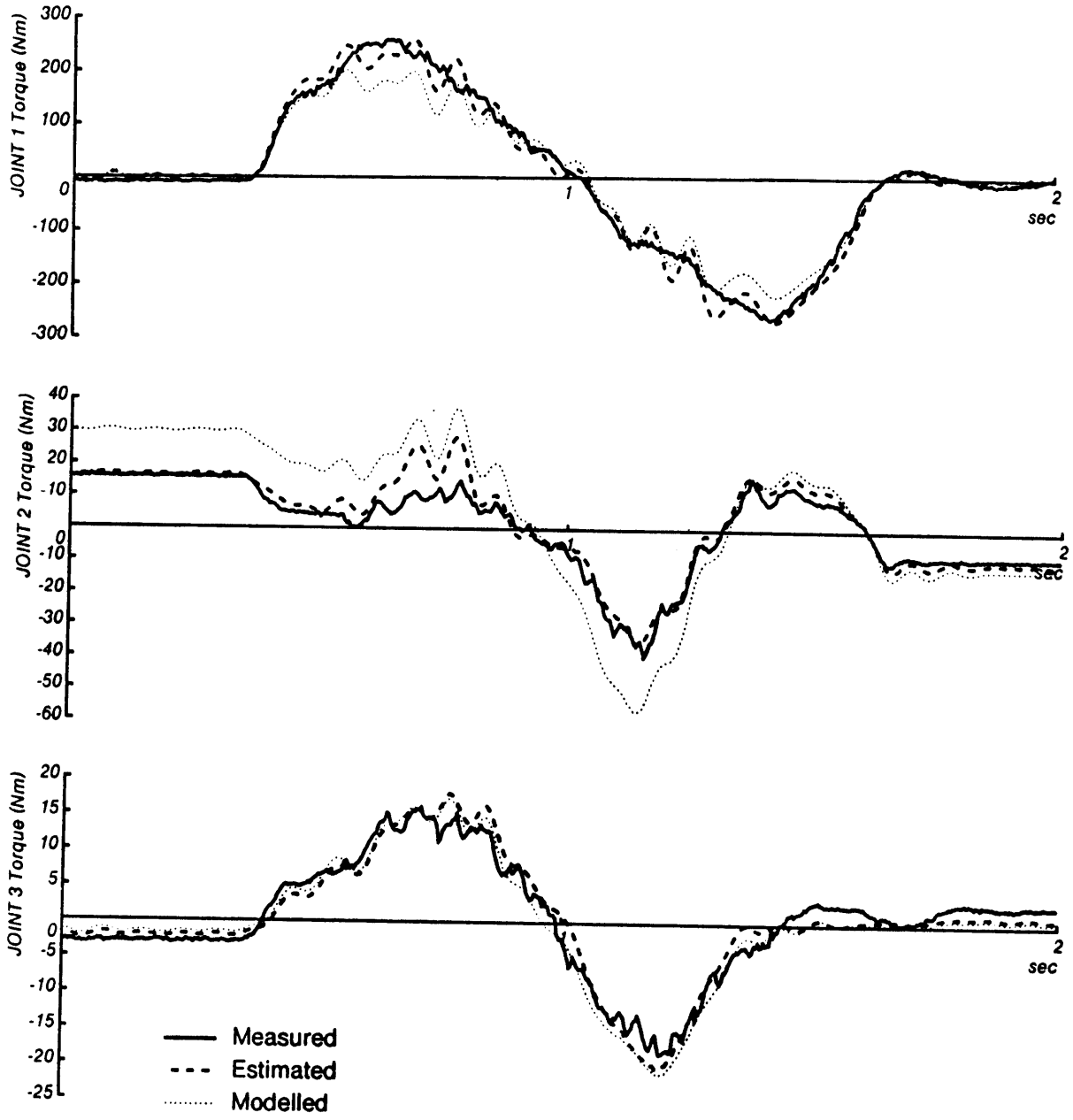

Figure 4: The measured, CAD-modelled, and the estimated joint torques

4 Discussion

Good estimates of the link inertial parameters were obtained, as determined from the match of predicted torques to measured torques. The potential advantage of this movement-based estimation procedure for increased accuracy as well as convenience was demonstrated by the less accurately predicted torques based on the CAD-modeled inertial parameters.

There are three groups of inertial parameters: fully identifiable, completely unidentifiable, and identifiable in linear combinations. Membership of a parameter in a group depends on the manipulator's particular geometry. Some link inertial parameters are unidentifiable because of restricted motion near the base and the lack of full force-torque sensing at each joint. For the first link, rotation is only possible about its z axis. Suppose full force-torque sensing is available at joint 1. It can be seen from (1) that I_{xz_1}, I_{xy_1}, and I_{yy_1} are unidentifiable because they have no effect on joint torque. Since the gravity vector is parallel to the z axis, c_{z_1} is also unidentifiable. If it is now supposed that only torque about the z axis can be sensed, then all inertial parameters for link 1 become unidentifiable except I_{zz_1}.

In a multi-link robot a new phenomenon arises. Some parameters can only be identified in linear combinations, because proximal joints must provide the torque sensing to identify fully the parameters of each link. Certain parameters from distal links are carried down to proximal links until a link appears with a rotation axis oriented appropriately for completing the identification. In between, these parameters appear in linear combinations with other parameters. This partial identifiability and the difficulty of analysis become worse as the number of links are increased.

The ridge regression automatically resolves the linear combinations in a least squares sense, which make these inertial parameters appear superficially different from those derived by CAD modeling. An alternative method is generation and examination of the closed-form dynamics, which is a complex procedure for more than two degrees of freedom.

A second alternative is a numerical analysis via singular value decomposition of $\mathbf{K}$ in (8), yielding (Golub and Van Loan, 1983)

$$\mathbf{K} = \mathbf{U}\Sigma\mathbf{V}^T$$

where $\Sigma = \text{diag}\{\sigma_i\}$ and $\mathbf{U}$ and $\mathbf{V}^T$ are orthogonal matrices. For each column of $\mathbf{V}$ there corresponds a singular value σ_i which if not zero indicates that linear combination of parameters, $\mathbf{v}_i^T\boldsymbol{\psi}$, is identifiable. Since $\mathbf{K}$ is a function only of the geometry of the arm and the commanded movement, it can be generated exactly by simulation rather than by actually moving the real arm and recording data with

inevitable noise.

The above two rocedures isolate several sets of parameters whose linear combinations within each set are identifiable. Then we can arbirtrarily set all but one of the parameters of each set to zero and apply the estimation algorithm for the reduced set of fully identifiable parameters. As shown in Table 2, for our 3 link manipulator, these two procedures result in grouping the 30 inertial parameters into the following categories:

1. fully identifiable: $m_3c_{x_3}$, $m_3c_{y_3}$, I_{xy_3}, I_{xz_3}, I_{yz_3}, I_{zz_3}, $m_2c_{x_2}$, I_{xy_2}, I_{xz_2}
2. completely unidentifiable: m_2, $m_2c_{z_2}$, m_1, $m_1c_{x_1}$, $m_1c_{y_1}$, $m_1c_{z_1}$, I_{xx_1}, I_{xy_1}, I_{xz_1}, I_{yy_1}, I_{yz_1}
3. identifiable in linear combinations: m_3, $m_3c_{z_3}$ I_{xx_3}, I_{yy_3}, $m_2c_{y_2}$, I_{xx_2}, I_{yy_2}, I_{yz_2} I_{zz_2}, I_{zz_1}

The completely unidentifiable parameters can be arbitrary set to zero. For the linear combination parameters, the combinations of these parameters that can be identified together are shown in table 3. To obtain a particular solution for these parameters, one can set $m_3, m_3c_{z_3}, I_{xx_3}$, and I_{xx_2} to zero. Then, the remaining 6 parameters of this category can be treated as fully identifiable parameters along with the 9 parameters in the first category. Those columns of $\mathbf{K}$ which correspond to the 15 zeroed parameters are taken out, reducing $\mathbf{K}^T\mathbf{K}$ to a 15×15 full rank matrix. Now, simple least squares can be applied to estimate the 15 identifiable parameters. For our implementation experiments, the results of this method agreed with the results of ridge regression presented in the previous section.

Although not as simple as the ridge regression method, these methods are attractive since it allows us to reformulate the dynamics in terms of only the identifiable parameters. This then can increase the efficiency of the corresponding inverse dynamics computation for controller implementations (Hollerbach and Sahar, 1983). We are still investigating these issues with eventual goals of studying the effectiveness of different control algorithms using the estimated dynamic model and analyzing their robustness with modelling errors.

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APPENDIX

Beginning with an n-joint manipulator, we assume each link i has a local coordinate system $\mathbf{P}_i$ with origin fixed at joint i; the vector $\mathbf{p}_i$ locates $\mathbf{P}_i$ with respect to a global coordinate system $\mathbf{O}$ (Figure 1). At the center of mass, a coordinate system $\mathbf{Q}_i$ is aligned with the principle axes of inertia. The center of mass is located with respect to $\mathbf{P}_i$ by the vector $\mathbf{c}_i$ and with respect to the global origin $\mathbf{O}$ by $\mathbf{q}_i$.

The inertial parameters of mass, center of mass, and moment of inertia are related to the motion of link i by the Newton-Euler equations:

$$ {}_q\mathbf{f}_i = \mathbf{f}_{ii} + m_i\mathbf{g} = m_i\ddot{\mathbf{q}}_i \tag{11} $$

$$ {}_q\mathbf{n}_i = \mathbf{n}_{ii} - \mathbf{c}_i \times \mathbf{f}_{ii} = {}_q\mathbf{I}_i\dot{\omega}_i + \omega_i \times ({}_q\mathbf{I}_i\omega_i) \tag{12} $$

where

${}_q\mathbf{f}_i =$ the net force acting on link i,

${}_q\mathbf{n}_i =$ the net torque about the center of mass of link i,

$\mathbf{f}_{ij} =$ the force at joint i due to motion of link j alone,

$\mathbf{n}_{ij} =$ the torque at joint i due to motion of link j alone,

$m_i =$ the mass of link i,

${}_q\mathbf{I}_i =$ the moment of inertia about the center of mass,

$\omega_i =$ the angular velocity vector, and

$\mathbf{g} =$ the gravity vector ($\mathbf{g} = [0, 0, -9.8\,\mathrm{m/sec^2}]$).

Although the location of the center of mass and hence its acceleration $\ddot{\mathbf{q}}_i$ are unknown, the latter is related to the acceleration $\ddot{\mathbf{p}}_i$ of joint i by (Symon, 1971):

$$ \ddot{\mathbf{q}}_i = \ddot{\mathbf{p}}_i + \dot{\omega}_i \times \mathbf{c}_i + \omega_i \times (\omega_i \times \mathbf{c}_i) \tag{13} $$

Substituting into (11), which then substitutes into (12),

$$ \mathbf{f}_{ii} = m_i(\ddot{\mathbf{p}}_i - \mathbf{g}) + \dot{\omega}_i \times m_i\mathbf{c}_i + \omega_i \times (\omega_i \times m_i\mathbf{c}_i) \tag{14} $$

$$ \begin{aligned} \mathbf{n}_{ii} = (\mathbf{g} - \ddot{\mathbf{p}}_i) \times m_i\mathbf{c}_i + {}_q\mathbf{I}_i\dot{\omega}_i + m\mathbf{c}_i \times (\dot{\omega}_i \times \mathbf{c}_i) \\ + \omega_i \times ({}_q\mathbf{I}_i\omega_i) + m_i\mathbf{c}_i \times (\omega_i \times (\omega_i \times \mathbf{c}_i)) \end{aligned} \tag{15} $$

Although the terms $\mathbf{c}_i \times (\dot{\omega}_i \times \mathbf{c}_i)$ and $\mathbf{c}_i \times (\omega_i \times (\omega_i \times \mathbf{c}_i))$ are quadratic in the unknown location of the center of mass $\mathbf{c}_i$, they may be eliminated by expressing

the moment of inertia tensor about the joint i origin ${}_p\mathbf{I}_i$ instead of about the center of mass ${}_q\mathbf{I}_i$. By the parallel axis theorem (Symon, 1971),

$$ {}_p\mathbf{I}_i = {}_q\mathbf{I}_i + m_i[(\mathbf{c}_i^T\mathbf{c}_i)\mathbf{1} - (\mathbf{c}_i\mathbf{c}_i^T)] \tag{16} $$

where $\mathbf{1}$ is the 3 dimensional identity matrix. Rewriting (15) and using (16),

$$ \begin{aligned} \mathbf{n}_{ii} &= (\mathbf{g} - \ddot{\mathbf{p}}_i) \times m_i\mathbf{c}_i + {}_q\mathbf{I}_i\dot{\omega}_i + m_i[(\mathbf{c}_i^T\mathbf{c}_i)\mathbf{1} - (\mathbf{c}_i\mathbf{c}_i^T)]\dot{\omega}_i \\ &\quad + \omega_i \times ({}_q\mathbf{I}_i\omega_i) + \omega_i \times (m_i[(\mathbf{c}_i^T\mathbf{c}_i)\mathbf{1} - (\mathbf{c}_i\mathbf{c}_i^T)]\omega_i) \\ &= (\mathbf{g} - \ddot{\mathbf{p}}_i) \times m_i\mathbf{c}_i + {}_p\mathbf{I}_i\dot{\omega}_i + \omega_i \times ({}_p\mathbf{I}_i\omega_i) \end{aligned} \tag{17} $$

The following notation aids formulating a system of linear equations from those above:

$$ \omega \times \mathbf{c} = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} \begin{bmatrix} c_x \\ c_y \\ c_z \end{bmatrix} \triangleq [\omega\times]\,\mathbf{c} $$

$$ \mathbf{I}\omega = \begin{bmatrix} \omega_x & \omega_y & \omega_z & 0 & 0 & 0 \\ 0 & \omega_x & 0 & \omega_y & \omega_z & 0 \\ 0 & 0 & \omega_x & 0 & \omega_y & \omega_z \end{bmatrix} \begin{bmatrix} I_{xx} \\ I_{xy} \\ I_{xz} \\ I_{yy} \\ I_{yz} \\ I_{zz} \end{bmatrix} \triangleq [\bullet\omega] \begin{bmatrix} I_{xx} \\ I_{xy} \\ I_{xz} \\ I_{yy} \\ I_{yz} \\ I_{zz} \end{bmatrix} $$

where

$$ \mathbf{I} = \mathbf{I}^T = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{xy} & I_{yy} & I_{yz} \\ I_{xz} & I_{yz} & I_{zz} \end{bmatrix} $$

A matrix equation can then be written from (14) and (17):

$$ \begin{bmatrix} \mathbf{f}_{ii} \\ \mathbf{n}_{ii} \end{bmatrix} = \begin{bmatrix} \ddot{\mathbf{p}}_i - \mathbf{g} & [\dot{\omega}_i\times] + [\omega_i\times][\omega_i\times] & \mathbf{0} \\ \mathbf{0} & [(\mathbf{g} - \ddot{\mathbf{p}}_i)\times] & [\bullet\dot{\omega}_i] + [\omega_i\times][\bullet\omega_i] \end{bmatrix} \begin{bmatrix} m_i \\ m_i c_{x_i} \\ m_i c_{y_i} \\ m_i c_{z_i} \\ I_{xx_i} \\ I_{xy_i} \\ I_{xz_i} \\ I_{yy_i} \\ I_{yz_i} \\ I_{zz_i} \end{bmatrix} $$