



LIBRARY of the MASSACHUSETTS INSTITUTE OF TECHNOLOGY Digitized by the Internet Archive in 2011 with funding from Boston Library Consortium Member Libraries

http://www.archive.org/details/incidenceofinter00diam

# INCIDENCE OF AN INTEREST INCOME TAX

BY

PETER A. DIAMOND

11:22.1	· .	TE,H.
NOV :	27	1967
DEVIEY	LID	RARY

Number 5 -- October 1967

Copy 2

This paper was partially supported by the National Science Foundation (grant number GS 1585). The views expressed in this paper are the author's sole responsibility, and do not reflect those of the National Science Foundation, the Department of Economics, nor of the Massachusetts Institute of Technology. HB31 .M415 no.5 c.2

-

RECEIVED MAY 8 1968 M. L. T. LIPPADISC

# INCIDENCE OF AN INTEREST INCOME TAX

Recent analyses of the corporation income tax have tended to concentrate either on short run shifting through market imperfections<sup>1/</sup> or on the shifting of capital between the incorporated and unincorporated sectors.<sup>2/</sup> Little attention has been paid to the impact of this tax on saving propensities and the level of capital formation. Similarly, analyses of the personal income tax have not fully explored the long run effect of continual taxation of the return to savings on the growth of an economy. As a partial filling of this gap, using a simple competitive growth model, this paper explores the differential incidence of an interest income tax rather than lump sum tax. Savings are assumed to satisfy maximization of individual lifetime utility functions while production conforms to an aggregate neoclassical production function. Individuals are presumed to live for two periods, working only in the first, with labor supplied inelastically.<sup>3/</sup>

During part of the work on this paper the author was an Overseas Fellow at Churchill College, Cambridge. He is indebted to Karl Shell for helpful discussions on this topic. Financial support was provided in part by the National Science Foundation under grant GS 1585. The author bears sole responsibility for errors and opinions.

\*

1/ See, e.g., M. Kryzyzaniak and R.A. Musgrave, <u>The Shifting of the Corporation</u> <u>Income Tax</u> (Baltimore, 1963), or R.J. Gordon, "The Incidence of the Corporation Income Tax in U.S. Manufacturing, 1925-62," <u>American Economic Review</u> (forthcoming), and the lengthy bibliographies contained there.

2/ See,e.g., A.C. Harberger, "The Corporation Income Tax: An Empirical Appraisal," in <u>Tax Revision Compendium</u>, U.S. House of Representatives, Ways and Means Committee (Washington, 1959).

3/ For a fuller treatment of this model the reader is referred to P.A. Diamond, "National Debt in a Neoclassical Growth Model," <u>American Economic Review</u>, December, 1965, (LV.6).

532212

It is seen that the differential incidence of an interest income tax is an increase in the gross of tax rate of return and decrease in the wage. Surprisingly, for low elasticities of substitution, the net of tax rate of return may also rise. Just comparing long run equilibria, it is seen that the interest income tax decreases the utility of a representative individual when the net interest rate exceeds the rate of growth.

The analysis is performed for the balanced budget incidence of an increase in the interest income tax used to finance lump-sum transfers to the same individuals paying the tax (the government budget staying balanced). This is equivalent to the differential incidence of an interest income tax increase and lump-sum tax decrease. Since labor is assumed to be supplied inelastically, analysis in this model of an income tax, falling on wages and interest, would not differ from the analysis performed.

## 1. TECHNOLOGY:

Each individual is assumed to live for two periods, but only to work during the first period of his life. Assuming that labor is supplied inelastically by each worker, the labor supply of the  $t^{\underline{th}}$  period equals the number of individuals born at the start of the  $t^{\underline{th}}$  period. It is assumed that the labor force grows geometrically.

(1) 
$$L_{t} = L_{o} (1 + n)^{t}$$

Production is assumed to satisfy an unchanging (twice continuously differentiable) neoclassical production function with constant returns, positive marginal products, and a diminishing marginal rate of substitution,

(2) 
$$Y_t = F(K_t, L_t) = L_t f(k_t),$$

where  $K_t$  is the capital stock and  $k_t$  capital per worker in the t<sup>th</sup> period.

The assumption of competition in the factor markets implies the existence of a factor price frontier relating the wage to the rate of return on capital,

(3) 
$$w_t = \phi(r_t), \phi' = -k_t$$

- 2 -

2. SAVINGS DECISIONS:

It is assumed that there are no bequests and no government ownership of capital. Therefore all savings are made by individuals in their first year of life to maximize lifetime utility. In their second year individuals dissave, consuming all their capital and accrued interest. Thus the aggregate capital stock equals the savings of the members of the generation that is two years old. We can state the savings decision as 4/

(4) Maximize  $U(e_1,e_2)$  subject to  $e_1 + \frac{e_2}{1+r\tau} = w + \frac{T}{1+r\tau}$ where  $e_1$  and  $e_2$  are first and second period consumptions, w, the wage; r, the return to savings;  $\tau$ , one minus the interest income tax rate; and T, the lump sum transfers. For an individual born in period t, the variables  $e_1^t$  and  $w_t$  are those of period t; while  $e_2^{t+1}$ ,  $r_{t+1}$ , and  $T_{t+1}$  are those of period t+1. Substituting for  $e_2$ in the maximization, we can eliminate the constraint and restate the maximization as

(5) Maximize 
$$U(e_1, (w-e_1)(1+r\tau)+T)$$
.

The first order condition for this maximization is

(6)  $U_1 - (1+r\tau) U_2 = 0.$ 

This equation implicitly defines optimal first year consumption as a function of w, r,  $\tau$ , and T, which we know can be written as a function of the budget constraint  $\hat{w}$  and the net rate of interest  $\hat{r}$ .

(7) 
$$e_1^* = e(w, r, \tau, T) = \hat{e}(\hat{w}, \hat{r}) = \hat{e}(w + \frac{T}{1+r\tau}, r\tau).$$

We assume that both present and future consumption are normal goods: i.e.

(8) 
$$0 \stackrel{\leq}{=} \frac{\partial \hat{e}}{\partial \hat{\omega}} \stackrel{\leq}{=} 1.$$

4/ It is assumed that U is twice continuously differentiable with positive marginal utility and a diminishing marginal rate of substitution.

Differentiating the first order condition, we can obtain the derivatives of the consumption function with respect to the tax variables in terms of the derivatives of the utility function.

(9) 
$$\frac{\partial e}{\partial \tau} = \frac{r U_2 + r(w - e_1) ((1 + r\tau) U_{22} - U_{12})}{U_{11} - 2(1 + r\tau) U_{12} + (1 + r\tau)^2 U_{22}}$$

(10) 
$$\frac{\partial e}{\partial T} = \frac{(1+r\tau)U_{22} - U_{12}}{U_{11} - 2(1+r\tau)U_{12} + (1+r\tau)^2 U_{22}}$$

If the tax variables are such that the individual receives as a lump sum exactly what is taken from him by the interest income tax, then the tax variables must satisfy

(11) 
$$T - (1-\tau) r (w-e_1^{\pi}) = 0$$

or

(12) 
$$T - (1-\tau) r (w-e(w,r,\tau,T)) = 0$$

Thus to maintain this relation, the changes in the tax variables must satisfy

(13) 
$$\frac{\partial T}{\partial \tau} = -\frac{r(w-e_1^*) + (1-\tau)r}{1 + (1-\tau)r} \frac{\partial e}{\partial \tau}}{\frac{\partial e}{\partial \tau}}$$

We can now calculate the change in savings arising from a tax rate and lump sum transfer change which leaves the government budget unchanged. We can call this the tax compensated change.<sup>5/</sup>

5/ Note that this differs from the Slutsky compensated change where utility, not tax revenue, is kept constant. The Slutsky substitution effect equals

$$\frac{rU_2}{U_{11} - 2(1+r_{\tau}) U_{12} + (1+r_{\tau})^2 U_{22}}$$

(14) 
$$\frac{\partial e_{1}^{\star}}{\partial \tau} = \frac{\partial e}{\partial \tau} + \frac{\partial e}{\partial T} \frac{\partial T}{\partial \tau}$$
$$= \frac{r U_{2}}{U_{11} - (1 + r \tau + 1 + r) U_{12} + (1 + r) (1 + r \tau) U_{22}}$$

The numerator of this expression is positive, since marginal utility is, while the denominator is negative by the assumption of normality of present and future consumption.<sup>6/</sup>

This analysis can also be seen geometrically and is portrayed in Figure 1. AA represents the budget line in the absence of taxes. The compensated equilibria for



6/ Constraint (8) can be written as

$$0 \stackrel{\leq}{=} \frac{-U_{12}(1+r\tau) + U_{22}(1+r\tau)^2}{U_{11} - 2(1+r\tau) U_{12} + (1+r\tau)^2 U_{22}} \stackrel{\leq}{=} 1.$$

Thus  $0 \stackrel{\geq}{=} - U_{12}(1+r\tau) + U_{22}(1+r\tau)^2$ ,  $0 \stackrel{\geq}{=} U_{11} - (1+r\tau) U_{12}$ . Multiplying the first inequality by  $\frac{1+r}{1+r\tau}$  and adding the two gives the needed inequality. various tax rates lies along this line. BB is the budget line for one particular tax rate and E the equilibrium point, II being the indifference curve through E (and tangent to BB). Normality of present and future consumption implies that indifference curves become steeper as we move along AA to the left. Thus we see that equilibria at higher tax rates result in higher optimal first period consumption.<sup>7/</sup> 3. EOUILIBRIUM:

There will be equilibrium in period t when the amount of savings forthcoming at the interest rate (and the previous wage) will lead to production resulting in the marginal product of capital equalling the gross of tax rate of interest. Noting that the labor force grows at n percent, the equilibrium capital-labor ratio satisfies

(15)

$$(1+n) k_{t+1} = (w_t - e (w_t, r_{t+1}, \tau_{t+1}, T_{t+1})),$$

where

•

(16)  $r_{t+1} = f'(k_{t+1}).$ 

Combining these two equations with the factor price frontier, we have a set of dynamic equations which describe the period by period equilibrium of the economy for a given initial position and time path of  $\tau$  and T which keep the government budget balanced. Before depicting this economy graphically we shall make two further assumptions: first, that for each tax rate the economy has a unique, stable long run equilibrium at a positive capital-labor ratio; and second, that an increased

<sup>7/</sup> We can also see that in the absence of the assumed increasing steepness of indifference curves there may be two equilibria at a given tax rate, with different transfer payments to have no revenue collected.

wage actually results in increased equilibrium savings.<sup>8/</sup> The first assumption implies that the discussion of asymptotic behavior and the comparisons of steady states are relevant. It is thus implied that in the neighborhood of an equilibrium point  $\left|\frac{\mathrm{d}\mathbf{r}_{t+1}}{\mathrm{d}\mathbf{r}_{t}}\right| \leq 1$ . By the assumption of normality of consumer demand, we know that a higher wage implies an increased supply of savings. However, if the demand for savings (defined by profit maximization and the production function) is steeper than the supply, the increased supply of savings will result in a decreased equilibrium quantity of savings, with the effect of the interest rate increase dominating that of the wage increase. This result seems strange and so is ruled out by assumption. Analysis of this case can be performed, although it is not in this paper. This second assumption is that  $\frac{\partial \mathbf{r}_{t+1}}{\partial \mathbf{w}_t} < 0$ . Since the factor price frontier is negatively sloped,  $\frac{\partial \mathbf{w}_t}{\partial \mathbf{r}_t} = \phi^* < 0$ , we have as a result of our two assumptions the condition

(17) 
$$0 \stackrel{\leq}{=} \frac{\partial^{2} \mathbf{r}_{t+1}}{\partial \mathbf{r}_{t}} \stackrel{\leq}{=} 1$$

in a neighborhood of an equilibrium point.

Plotting the factor price frontier and the curve (denoted by  $\psi$ ) obtained by eliminating  $k_{t+1}$  from equations (15) and (16),

(18) 
$$r_{t+1} = f'((1+n)^{-1} (w_t - e(w_t, r_{t+1}, \tau_{t+1}, T_{t+1})))$$

(with either  $\tau_t$  or  $T_t$  fixed over time and the other defined by (12)), we can describe the economy's behavior over time.

8/

Both assumptions are discussed further in Diamond, op.cit.



# Figure 12

For an initial pair of values  $(w_1, r_1)$ , we can read subsequent one period equilibrium points from the  $\phi$  curve. The point E denotes the long run equilibrium.

For later use, let us now calculate the two restrictions on the nature of the equilibrium in terms of the parameters of the production and consumption functions. We shall do this for a constant tax rate.<sup>9/</sup> We can eliminate  $T_{t+1}$  by use of

(19)  $T_{t+1} = (1-\tau)r_{t+1}(w_t - e_1^{*t}) = (1-\tau)r_{t+1}(1+n)k_{t+1} = (1+n)(1-\tau)r_{t+1}f'^{-1}(r_{t+1})$ , where  $f'^{-1}$  is the inverse function relating the capital-labor ratio to the marginal product of capital. Combining equations (18) and (19) we have

20) 
$$r_{t+1} = f'((1+n)^{-1}(w_t - e(w_t, r_{t+1}, \tau, (1+n)(1-\tau)r_{t+1}f'^{-1}(r_{t+1}))))$$

9/ Thus we are analyzing the differential incidence of an interest income tax, rather than lump sum tax, which raises a varying amount of revenue over time with a fixed tax rate. C. Metcalf has pointed out that one can alternately perform differential incidence for the tax rate necessary to raise a given revenue. For this analysis, T would be fixed and  $\tau$  variable. Differentiating equation (20) implicitly we can rewrite the constraint as

(21) 
$$0 \stackrel{\geq}{=} \frac{\partial \mathbf{r}_{t+1}}{\partial \mathbf{w}_{t}} = \frac{1 - \frac{\partial \mathbf{e}}{\partial \mathbf{w}}}{(1+n)\mathbf{f'}^{-1} + \frac{\partial \mathbf{e}}{\partial \mathbf{r}} + \frac{\partial \mathbf{e}}{\partial \mathbf{T}}(1+n)(1-\tau)(\mathbf{f'}^{-1} + \mathbf{r}\mathbf{f''}^{-1})}.$$

Since  $\frac{\partial \mathbf{r}_{t+1}}{\partial \mathbf{r}_t} = \frac{\partial \mathbf{r}_{t+1}}{\partial \mathbf{w}_t} \frac{\partial \mathbf{w}_t}{\partial \mathbf{r}_t} = -k_t \frac{\partial \mathbf{r}_{t+1}}{\partial \mathbf{w}_t}$ , we can express the stability

constraint in the nieghborhood of an equilibrium as

(22) 
$$0 \leq \frac{\partial \mathbf{r}_{t+1}}{\partial \mathbf{r}_{t}} = \frac{-k_{t}(1-\frac{\partial \mathbf{e}}{\partial w})}{(1+n)f''^{-1}+\frac{\partial \mathbf{e}}{\partial \mathbf{r}}+\frac{\partial \mathbf{e}}{\partial T}(1+n)(1-\tau)(f'^{-1}+rf''^{-1})} \leq 1.$$

## 4. TAX RATE CHANGE:

We can now examine the impact of a once for all change in the tax rate. (We shall assume that the economy was in long run equilibrium at the time of the tax change, although this is not necessary for the analysis). In Figure 1, the  $\phi$  curve is unaltered by a change in the tax rate, but the  $\psi$  curve shifts. To determine the direction of the shift, we can take the partial derivative of  $r_{t+1}$  with respect to  $\tau$  in (20), holding w<sub>t</sub> constant,

(23) 
$$\frac{\partial \mathbf{r}_{t+1}}{\partial \tau} = \frac{-\left(\frac{\partial \mathbf{e}}{\partial \tau} - \frac{\partial \mathbf{e}}{\partial T}\right) (1+n) \mathbf{r}^{-1}}{(1+n)\mathbf{f}^{-1} + \frac{\partial \mathbf{e}}{\partial \mathbf{r}} + \frac{\partial \mathbf{e}}{\partial T}} (1+n)(1-\tau)(\mathbf{f}^{-1}+\mathbf{r}\mathbf{f}^{+})^{-1}}.$$

From equation (21) we know that the denominator of this expression is negative. The term in parentheses in the numerator is negative, as can be seen from the discussion of consumer response to taxation. (Substituting from (13) in (14) we obtain the term above divided by  $(1+(1-\tau)r \frac{\partial e}{\partial T})$ , which is positive). Thus the derivative in (23) is negative, implying that an increase in the tax rate (decrease in  $\tau$ ) shifts the  $\psi$  curve to the right at every wage.

As depicted in Figure 2, a once-for-all increase in the tax rate moves the economy from the long run equilibrium factor payments  $(w_0, r_0)$  to a series of period equilibria  $(w_1, r_1)$ ,  $(w_2, r_2)$ , ..., converging to the new long run equilibrium  $(w^*, r^*)$ .



Figure 2

In each period the wage is lower and the gross of tax interest rate higher than it would have been with a lower tax rate.

# 5. LONG-RUN EQUILIBRIUM:

We can calculate the change in long run equilibrium arising from the tax rate change and determine not only the change in factor payments, but also the change in the utility of a representative man living in long-run equilibrium. Taking equation (20), relating  $r_{t+1}$  to  $w_t$  and substituting for  $w_t$  by the factor price frontier, we have a difference equation giving the time path of the interest rate. Dropping time subscripts, we have a locus of long run equilibria for different tax rates.

(24) 
$$\mathbf{r} = \mathbf{f}'((1+n)^{-1}(\phi(\mathbf{r}) - e(\phi(\mathbf{r}), \mathbf{r}, \tau, (1+n)(1-\tau)\mathbf{r}\mathbf{f}'^{-1}(\mathbf{r}))).$$

We can implicitly differentiate this equation to obtain the asymptotic change in the interest rate from a change in the tax rate

(25) 
$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}\tau} = \frac{-(\frac{\partial e}{\partial \tau} - \frac{\partial e}{\partial T}(1+n)\mathbf{r}\mathbf{f'}^{-1}(\mathbf{r}))}{(1+n)\mathbf{f''}^{-1} + (\phi'(\mathbf{r})\frac{\partial e}{\partial w} + \frac{\partial e}{\partial r} + \frac{\partial e}{\partial T}(1+n)(1-\tau)(\mathbf{f'}^{-1}(\mathbf{r})+\mathbf{r}\mathbf{f''}^{-1}(\mathbf{r}))) - \phi'(\mathbf{r})}$$

Evaluating the stability constraint, (22), at the equilibrium point we see that the donominator of this expression is negative. As above, the numerator can be seen to be positive from the assumptions on consumer behavior. Thus, the assumptions underlying Figure 2 lead to the result depicted there, that an increase in the tax rate increases the asymptotic gross of tax interest rate and decreases the wage.

Thus, we have a decrease in wage and increase in tax rate, but an increase in interest rate. It is thus necessary to calculate the change in utility for an individual living in long-run equilibrium to determine its sign. (The next section will deal with the change in the net of tax interest rate.)

Writing utility as in (5) above,  $U(e_1,(w-e_1)(1+r,\tau) + T)$ , we can differentiate utility totally with respect to the tax rate.

(26) 
$$\frac{dU}{d\tau} = U_1 \frac{de_1}{d\tau} - U_2 (1+r.\tau) \frac{de_1}{d\tau} + U_2 (1+r.\tau) \frac{dw}{d\tau} + (w-e_1)(r + \tau \frac{dr}{d\tau}) + \frac{dT}{d\tau})$$

By the first order condition for utility maximization, the first two terms of this derivative sum to zero. Noting that

(27) 
$$\frac{dw}{d\tau} = -k \frac{dr}{d\tau} = -\frac{(w-e_1)}{(1+n)} \frac{dr}{d\tau};$$
$$\frac{dT}{d\tau} = -r(w-e_1) + (1-\tau) \left( (w-e_1) \frac{dr}{d\tau} + r \frac{d(w-e_1)}{d\tau} \right);$$

and

$$\frac{d(w-e_1)}{d\tau} = (1+n) \frac{dk}{d\tau} = (1+n) f''^{-1} \frac{dr}{d\tau}$$

we can rewrite the utility derivative as

(28) 
$$\frac{dU}{d\tau} = U_2 \left( \frac{(w-e_1)}{(1+n)} (n-r\tau) + (1+n)(1-\tau)r f''^{-1} \right) \frac{dr}{d\tau}$$

When the net of tax interest rate exceeds the rate of growth of the labor force, the expression in parenthesis is unambiguously negative, and thus the derivative is positive. Thus an increase in the tax rate decreases the utility of an individual living in long run equilibrium. The tax change has two impacts on the economy. By widening the divergence between marginal rates of substitution and transformation, there is increased inefficiency and a fall in utility. The tax also decreases capital formation. When savings were not previously excessive, this redistributes income toward the short run (where saving decreases) and away from the long run. However, if the economy were saving previously to a point where the rate of labor force growth exceeded the rate of interest for consumer decisions, there would have been long run inefficiency had the economy maintained that savings rate forever.<sup>10/</sup> Thus the decreased savings decreases this inefficiency and, in this case, tends to offset the decrease in utility arising from the impact of the increased tax rate on short run efficiency.

With these two elements (tax inefficiency and intertemporal redistribution) occurring simultaneously, the total change in utility of an individual living in long run equilibrium does not correspond to the formulas giving the impact of taxes on utility in a static setting.<sup>11/</sup>

10/

For a discussion of this sort of inefficiency see Diamond, op.cit.

11/ See, e.g., A. C. Harberger, "Taxation, Resource Allocation, and Welfare," in <u>The Role of Direct and Indirect Taxes in the Federal Revenue System</u> (A Conference Report of the NBER and the Brookings Institution) Princeton, 1964.

- 12 -

### 6. NET OF TAX INTEREST RATE:

We calculated above that the gross of tax interest rate necessarily rises with an increase in the tax rate. (In terms of just interest rates one would then say that part of the tax was shifted). Moreover, we shall now see that the net of tax interest rate may either fall or rise from one long run equilibrium to another. (Again, in terms of the rate of return, shifting may exceed one hundred percent;) Calculating the derivative across steady states we have

(29) 
$$\frac{d(r\tau)}{d\tau} = \tau \frac{dr}{d\tau} + r.$$

Substituting from equation (25) we have

(30) 
$$\frac{d(r\tau)}{d\tau} = \frac{(rk)(1+r\tau)^{-1}(1+r\tau+(n-r\tau)\frac{\partial e}{\partial w} + (\frac{1+n}{kf'})(1+r\tau+(r-r\tau)\frac{\partial e}{\partial w}))}{(1+n)f''^{-1} - (\phi'(r)(1-\frac{\partial e}{\partial w}) - \frac{\partial e}{\partial r} - \frac{\partial e}{\partial T}(1+n)(1-\tau)(f'^{-1}+rf''^{-1}))}$$

As with equation (25) the denominator of this expression is negative. The numerator will be signed by the inequality between kf'' and the other terms in the parentheses. Thus

(31) 
$$\frac{d(r\tau)}{d\tau} \left\{ \stackrel{>}{=} \right\} 0 \text{ as } - kf'' \left\{ \stackrel{\leq}{=} \right\} (1+n) \left( \frac{1+r\tau+(r-r\tau)\frac{\partial e}{\partial w}}{1+r\tau+(n-r\tau)\frac{\partial e}{\partial w}} \right)$$

Examining the expression on the right, the critical value of -kf'' (call it  $\zeta$ ) at which  $\frac{dr\tau}{d\tau}$  is zero must lie in the range (the limits are reversed for r<n):

(32)  $1 + n \stackrel{<}{=} \zeta \stackrel{<}{=} 1 + r$ .

Thus the sign of the response of the net of tax interest rate to a tax change depends primarily, but not exclusively, on the shape of the production function (and the position of equilibrium). Since the elasticity of substitution equals  $-\frac{f'(f-kf')}{kff''}$  we see that the net of tax interest rate will tend to rise with a tax increase for low elasticities while falling for high elasticities.<sup>12/</sup>

12/ Trying some arbitrary numbers, with a share of labor of 2/3  $\left(\frac{f-kf'}{f}\right)$ , a rate of return of .2, and a rate of labor force growth of .05, the critical value of the elasticity lies between .11 and .13. Hopefully the use of annual numbers in a model with much longer periods is not too misleading.

Let us briefly consider two special cases. With a Cobb Douglas production function we have:

(33) 
$$y = f(k) = Ak^{\alpha}$$
$$r = f'(k) = \alpha Ak^{\alpha-1}$$
$$f''(k) = \alpha(\alpha-1)Ak^{\alpha-2}$$

Thus we see that

(34) 
$$-kf^{\dagger} = \alpha(1-\alpha)Ak^{\alpha-1} = (1-\alpha)r.$$

With a growing labor force, and rate of return on capital below 100 percent, the net rate of interest falls for a tax increase. With a marginal propensity to consume sufficiently close to one, even the latter assumption is not needed for the conclusion.

To consider the case of fixed coefficients we must drop equation (16), relating the rate of return to the capital stock. Equation (15), with  $k_{t+1}$  now a constant determines the interest rate which will bring forth sufficient savings for full employment of both factors. Further the wage is related to the interest rate, not by the factor price frontier, but by exhaustion of the product

(35) w + rk = y.

Thus the locus of long run equilibria can be written

(36) 
$$k(1+n) = y - rk - e(y-rk, r, \tau, (1-\tau)rk (1+n)).$$

Thus, differentiating implicitly

(37) 
$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}\tau} = \frac{-\left(\frac{\partial \mathbf{e}}{\partial \tau} - \frac{\partial \mathbf{e}}{\partial T} \mathbf{r} \mathbf{k}(1+\mathbf{n})\right)}{\mathbf{k}\left(1 - \frac{\partial \mathbf{e}}{\partial w}\right) + \frac{\partial \mathbf{e}}{\partial \mathbf{r}} + \frac{\partial \mathbf{e}}{\partial T}(1-\tau)\mathbf{k}(1+\mathbf{n})};$$
  
(38) 
$$\frac{\mathrm{d}\mathbf{r}_{\tau}}{\mathrm{d}\tau} = \frac{\mathbf{r}\mathbf{k}\left(1 - \frac{\partial \mathbf{e}}{\partial w} + \frac{\partial \mathbf{e}}{\partial w} \left(\frac{1+\mathbf{n}}{1+\mathbf{r}\tau}\right)\right)}{\mathbf{k}\left(1 - \frac{\partial \mathbf{e}}{\partial w}\right) + \frac{\partial \mathbf{e}}{\partial \mathbf{r}} + \frac{\partial \mathbf{e}}{\partial T}(1-\tau)\mathbf{k}(1+\mathbf{n})}.$$

The denominator of this expression is negative, while the numerator is positive. Thus with fixed coefficients a tax increase increases the net of tex interest rate. This result can be seen by examining behavior of savers, keeping in mind that a fixed amount of savings is necessary to achieve equilibrium. With the tax increase, savings are decreased at the same wage and interest rate, as we saw above. Raising the net of tax interest rate to the previous level still leaves first period consumption too high, because of the effect of the increased transfer payment arising from the tax rate increase. Thus even in the first period after a tax rise, the net of tax interest rate rises. A fortiori, with a declining wage overtime, it must rise further to induce adequate savings.

### 7. CONCLUSION:

Investigation of a simple competitive model has shown that the differential incidence of an interest income tax rather than a lump sum tax raises the gross interest rate and lowers the wage. The net of tax interest rate may rise or fall but tends to rise for low elasticities of substitution, falling for high ones. The movement of gross of tax relative shares will depend on the elasticity of substitution as the capital-labor ratio decreases as a result of a tax increase. Comparing stationary states, the utility of a representative man falls with a tax increase. While this model does not pretend to represent a model of the U.S. economy today, it does suggest that effects of taxation on saving are sizeable and may disrupt attempts to measure short run shifting by time series analysis.

- 15 -





[.1. co c

