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PORTFOLIO CHOICE, INVESTMENT, AND GROWTH

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August 1968

Working Paper 24

This research was supported
partially by grants to the
authors from the National
Science Foundation.

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I. Introduction

One of the basic postulates of monetary theory and policy is that an increase in the money supply raises income by exerting at least temporary downward pressure on the rate of interest. Since it is generally believed that the savings rate is rather unresponsive to changes in the rate of interest, the key to this postulate, and to the belief that monetary policy can affect aggregate demand and its composition, seems to be the existence of a negative relationship between the demand for investment goods and the market rate of interest.

The micro foundations of such a relationship, however, are very weak. It is well known from the work on investment theory by Haavelmo³ and others that an analysis of profit maximizing behavior on the part of firms can at most offer a relationship between the desired stock of capital and the rate of interest, but not a relationship between the desired rate of increase in the stock of capital and the rate of interest. The demand for investment cannot be derived from the demand for capital. Demand for a finite addition to the stock of capital can lead to any rate of investment.

¹ A preliminary version of this paper was presented at the Conference on Money and Economic Growth held at Brown University, June 1968.

² We would like to acknowledge the substantial contribution of Stanley Fischer to this paper. Mr. Fischer participated in many of our preliminary conversations about its substance and shape, and undertook extensive editorial work in the later stages. We would also like to thank J. Mirrlees, D. Patinkin and C. von Weizsäcker for our very helpful discussions with them on the subject of this paper.

³ Trygve Haavelmo, A Study in the Theory of Investment, Chicago, University of Press, 1960.

Recent attempts to derive a marginal efficiency of investment schedule from a firm's profit maximizing behavior have mostly relied on the assumption that there are costs to adjusting the actual to the desired stock of capital at a fast rate; that is, the additions to the capital stock which result from each additional dollar spent on investment diminishes with the level of investment.¹ Alternatively, they have simply assumed that once a firm finds out what its optimum stock of capital is (if it exists), it then adjusts the actual to the desired stock with a lag,² the most simple of these models being the one in which the rate of investment is a linear function of the difference between the desired and the actual stock of capital.

In some models of economic growth this theory requires a permanent gap between the desired and the actual stocks of capital in a state of steady growth to generate the investment necessary to maintain a constant capital-labor ratio over time when population is growing.^{3,4}

¹For this approach see R. Eisner and R.H. Strotz, "Determinants of Business Investment," Research Study Two in Impacts of Monetary Policy, Englewood Cliffs, N.J., Prentice Hall, 1963; J.P. Gould, "Market Value and the Theory of Investment of the Firm," American Economic Review, Vol. LVII, No. 4 (Sept. 1967), 910-913; and Arthur Treadway, "Rational Entrepreneurial Behavior and the Dynamics of Investment," unpublished Ph.D. dissertation submitted to the University of Chicago, March 1967.

²Dale W. Jorgenson, "Capital Theory and Investment Policy," American Economic Review, Vol. LIII, No. 2 (May 1963), 247-59.

³We are not asserting here that disequilibrium theories are uninteresting or irrelevant; however, since we know so little about behavior in conditions of disequilibrium, it is advisable to consider whether any phenomenon cannot be adequately explained by an equilibrium theory. We show that investment can be explained in this way. Probably, there are phenomena (such as unemployment) which can only be explained by disequilibrium theories.

⁴Richard R. Nelson, "Full Employment Policy and Economic Growth," American Economic Review, Vol. LVI, No. 5 (Dec. 1966), 1178-1192.

Does this mean that a theory of economic growth that wants to explain the existence of a state of steady growth with positive investment has to rely on a disequilibrium theory of investment or on the existence of continuous technical change? Not at all. In this paper we present a very simple model of growth based on an equilibrium theory of investment. The level of investment is determined jointly by the interaction of the stock of capital, its demand, which is treated as an integral part of the demand for assets by wealth owners, and the flow supply of investment goods as determined by the producers of capital goods. Even if the price of capital is such that the marginal return to the existing stock of capital makes the stock instantaneously optimal, this does not necessarily imply that this price makes it optimal for producers of capital goods not to produce any capital goods at all.

We believe that our model of investment corresponds quite closely to Keynes' vision of the investment process. In his response to four comments on the General Theory,¹ he states²

The owner of wealth, who has been induced not to hold his wealth in the shape of hoarded money, still has two alternatives between which to choose. He can lend his money at the current rate of money-interest or he can purchase some kind of capital-asset. Clearly in equilibrium these two alternatives must offer an equal advantage to the marginal investor in each of them. This is brought about by shifts in the money-prices of capital-assets relative to the prices of money-loans. The prices of capital-assets move until, having regard to their prospective yields and account being taken of all those elements of doubt and uncertainty, interested and disinterested advice, fashion, convention, and what else you will, which affect the mind of the investor, they offer an equal apparent advantage to the marginal investor who is wavering between one kind of investment and another.

¹J.M. Keynes, "The General Theory of Employment," Quarterly Journal of Economics (February 1937), reprinted in The New Economics, Seymour E. Harris, ed., New York, Alfred A. Knopf, 1948, 181-193. The page reference is to this latter source.

²p. 188.

...Capital-assets are capable, in general, of being newly produced. The scale on which they are produced depends, of course, on the relation between their costs of production and the prices which they are expected to realize in the market. Thus if the level of the rate of interest taken in conjunction with opinions about their prospective yield raise the prices of capital-assets, the volume of current investment (meaning by this the value of the output of newly-produced capital-assets) will be increased; while if, on the other hand, these influences reduce the prices of capital-assets, the volume of current investment will be diminished.

Given the level of technology and the stock of capital which is a result of past saving, the only variable that is unequivocally related to the level of investment is the price of capital goods. It is this emphasis, together with the notion of instantaneous stock equilibrium in the assets markets determining asset prices and yields, to be discussed below, which we share with Keynes' description quoted above. When the price of capital goods is high, the level of investment is also high; while if the price of capital is low, investment is also low. This would seem surprising if we look at the investment function as an ex-ante demand relationship because one would then think that the higher is the price of capital, the lower will be the level of investment. This result is not at all surprising if we think of the investment function as an ex-post supply relationship. The higher is the relative price of capital, the larger will be the share of the economy's total resources that producers will find profitable to allocate to the production of investment goods, and therefore, the higher will be the rate of capital accumulation.

We do not, however, assume the existence of a negative relationship between the desired level of investment and the market rate of interest and we show that such a relationship may not even exist between the actual, ex-post, rate of investment and the market rate of interest.¹ Other things equal, an increase in the rate of interest lowers the demand for capital on the part of wealth owners,

¹J.G. Witte, "The Microfoundations of the Social Investment Function," Journal of Political Economy, Vol. 71, No. 6 (October 1963), 441-56, takes a similar approach to ours but concludes with an ex-post relationship between the rate of investment and the market rate of interest.

decreasing on this account the equilibrium price of capital and thus lowering the optimum level of output in the investment goods sector. Other things, however, are not usually equal. The rate of interest is itself an endogenous variable of the system. A change in the rate of interest implies that some other variable in the system has changed, and the change in this variable may have increased the demand for capital that resulted from the increase in the rate of interest.

Although we conclude that there may be neither an ex-ante nor an ex-post negative relationship between the level of investment and the market rate of interest, this does not imply that the effects of monetary policy on aggregate demand or its composition are unpredictable. On the contrary, we shall prove that monetary policy is able to change aggregate demand and its composition by affecting the price of capital at which wealth owners are willing to hold the existing stock of capital. Monetary and fiscal policy jointly not only are able to affect the rate of growth by changing the relative price of capital, but they can also succeed in maintaining a constant price level while at the same time determining the economy's rate of growth.

Once the equilibrium level of investment is determined, we go on to consider the process by which capital and other assets find room in private portfolios. Instantaneous market equilibrium ensures that, given the state of expectations, wealth owners are willing to hold the existing stocks of assets and that their desired and actual savings are equal. It does not, however, guarantee that at the market clearing prices the value of the addition to each of their assets is equal to the value of the desired additions. That is, our equilibrium theory of investment does not assure the equality of the flow supplies and demand for each of the assets at the current equilibrium prices. Since markets clear at each moment of time, the process of wealth accumulation will generally require prices to change over time. Actual price changes, and

expected price changes, affect demand and supply schedules which in turn modifies equilibrium prices over time.

A number of different assumptions about expectations are popular. Among these are the static expectations hypothesis, where the present price level is always expected to remain in force; the perfect foresight hypothesis in which expectations are assumed to be always fulfilled; and several models of adaptive expectations, where the expectations can be wrong but the individual learns from his mistakes.¹ While static expectations models are too naive, particularly in dynamic systems where prices do change continuously over time, perfect foresight imposes severe restrictions on the solution path of the system and eliminates the possibility of many types of discretionary government policy. Adaptive expectations and some other models do leave room for discretionary policy and lead to "reasonable" results, but do not constitute an entirely satisfactory explanation of the process by which expectations are formed.²

In what follows we study the full dynamic behavior of the economy over time using a particular expectations hypothesis which has been employed previously by other authors. In this model we show how actual and expected price changes interact while portfolios are adjusted to make room for the newly supplied assets. In the context of this model, we also study how monetary and fiscal policy are able to stabilize aggregate demand so as to maintain a constant price level and how changes in the mix of fiscal and monetary policy affect the economy's long-run capital stock by modifying the price of

¹A version of the adaptive expectation hypothesis has been used by one of the authors in an earlier growth model. See Miguel Sidrauski, "Inflation and Economic Growth," Journal of Political Economy, Vol. 75, No. 6 (December 1967), 796-810.

²For some discussion of this, see John F. Muth, "Rational Expectations and the Theory of Price Movements," Econometrica, Vol. 29, No. 3 (July 1961), 315-35.

capital at which wealth owners are willing to hold the existing stock of capital at any point in time.

II. The Demand for Capital Services

Language itself presents several traps in thinking about investment, especially confusion between the demand for capital, the demand for capital services, and the demand for investment. Everyone has the notion that these ~~these~~ three things are related, and some people tend to talk as if two or more were identical. It seems to be appropriate to sort out these concepts at the start.

The simplest notion is the demand for capital services. This is analagous to the demand for the services of any factor of production, and arises indirectly from the demand for output. Capital services have the dimension of a rate or flow: the use of such and such a machine or building for one hour, week or month. The price which is determined in the market for capital services we will call the rental rate. It is measured in value units per hour, month or year, like the wage rate.

Producers will demand at any wage-rental rate combination, those amounts of capital and labor services which will equate their marginal products of capital and labor to those rates. In thinking about equilibrium in the market for capital services, we are free to ignore all matters concerned with the ownership of capital. A producer trying to equate marginal products to the prices of factor services is making a decision which is entirely divorced conceptually from considerations of owning capital. It may be that taxes and other market imperfections distort producer choices but in principle the producer in deciding his production plan does not care who owns the capital. He decides entirely on the basis of the market rental rate, which will be the same whether he, his competitor, or the King of Siam owns the machines and buildings.

If market imperfections prevent a producer from freely hiring capital services, then he should charge himself an implicit rental or shadow price equal

to the marginal product of the capital he is employing. The divergence of this shadow price from other producers' shadow rentals or from the average rental is a signal to the producer to adjust his capital stock. This gives us a kind of motive for individual firms to invest or disinvest if that is the only way they have of adjusting their employment of capital services. But it is not satisfactory as a theory of investment for two reasons. First, since capital of any type is assumed homogeneous and shiftable between different firms, the net desired adjustment of all firms is zero because there will be as many firms with low shadow prices who want to sell as there are with high shadow prices who want to buy. Second, even for an individual firm, the desired rate of adjustment is indeterminate. In fact, the faster the better. It seems unattractive to make the whole adjustment theory depend on some formulation of adjustment costs, as this account of the investment process requires.

In order to make clear the distinctions we seek to emphasize, we can use a two-sector model of production. One sector produces investment goods, I, and the other produces consumption goods, C. We take the consumption good to be the numeraire and denote the consumption price of capital goods by p_k .

Output in each industry is produced with the services of capital and labor, which are assumed to be proportional to the stocks employed. In this model, capital and labor are assumed homogeneous and perfectly mobile between sectors. We thus have:

$$(2.1) \quad Q_C = F_C(K_C, N_C)$$

$$(2.2) \quad Q_I = F_I(K_I, N_I)$$

where K_C and N_C are the amounts of capital and labor used in the production of C, and K_I and N_I are the amounts used in the production of I. Since we assume the production functions to be linearly homogeneous, per-capita output of consumption and capital goods is given by:

$$(2.3) \quad \frac{C}{N} = q_C = \frac{N_C}{N} \cdot F_C\left(\frac{K_C}{N_C}, 1\right) = \alpha_C f_C(k_C)$$

$$(2.4) \quad \frac{I}{N} = q_I = \frac{N_I}{N} \cdot F_I\left(\frac{K_I}{N_I}, 1\right) = \alpha_I f_I(k_I)$$

where

$$(2.5) \quad \alpha_C = \frac{N_C}{N}, \quad \alpha_I = \frac{N_I}{N}, \quad k_C = \frac{K_C}{N_C}, \quad k_I = \frac{K_I}{N_I}$$

Now the demand for the services of capital and labor will be determined by the familiar marginal productivity conditions, which bear no relationship to the ownership of factors. The rentals to capital in the two sectors under competitive conditions are the marginal value products of capital:

$$(2.6) \quad r_C = f'_C(k_C) = r$$

$$(2.7) \quad r_I = p_k f'_I(k_I)$$

If, as assumed, capital is perfectly mobile between the two sectors, these two must be equal at each moment of time:¹

$$(2.8) \quad f'_C(k_C) = p_k f'_I(k_I)$$

Similarly, the real wages in the two sectors under perfect competition are

¹Here, and for the rest of this paper, we neglect cases of specialization. In fact, the conclusions presented later in this paper carry through under such conditions. In the diagrams we use, \bar{p}_k is the price at which specialization to investment goods takes place, and \underline{p}_k is the consumption goods specialization price.

$$(2.9) \quad w_C = [f_C(k_C) - f'_C(k_C) k_C] = w$$

$$(2.10) \quad w_I = p_k [f_I(k_I) - f'_I(k_I) k_I]$$

With perfect labor mobility, these two must also be equal at each moment of time

$$(2.11) \quad [f_C(k_C) - f'_C(k_C) k_C] = p_k [f_I(k_I) - f'_I(k_I) k_I]$$

Since there is no alternative use for the services of the existing stock of capital, it is reasonable to assume that these services are supplied inelastically. We also assume that labor services are supplied inelastically; this is less plausible, but we do not want to study the working of the labor market in detail here.

If there is full employment of both factors, then

$$(2.12) \quad N_C + N_I = N \quad \text{or} \quad \alpha_C + \alpha_I = 1$$

and

$$(2.13) \quad K_C + K_I = K \quad \text{or} \quad \frac{K_C}{N_C} \cdot \frac{N_C}{N} + \frac{K_I}{N_I} \cdot \frac{N_I}{N} = \frac{K}{N} = k = k_C \alpha_C + k_I \alpha_I$$

Equations (2.3), (2.4), (2.8), (2.11), (2.12) and (2.13) are a system of six equations in six unknowns, k_C , k_I , α_C , α_I , q_C and q_I ; therefore, given p_k and k , we can determine these values.

We can then summarize the supply functions for consumption goods and newly produced capital in per-capita units as

$$(2.14) \quad q_C = q_C(k, p_k)$$

$$(2.15) \quad q_I = q_I(k, p_k)$$

These supply relations are pictured in Figure 2-1 where the production possibilities curve and the price line corresponding to p_{k_0} are shown.

The per-capita stock of capital, k , determines the production possibility locus, and the price of capital determines the allocation of resources between the production of capital and consumption goods.

An increase in the price of capital increases output in the investment goods sector. From Fig. 2-1, we see that as p_k increases from p_{k_0} to p_{k_1} , output of investment goods increases from q_{I_0} to q_{I_1} . Hence

$$(2.16) \quad \frac{\partial q_C}{\partial p_k} < 0 \qquad \frac{\partial q_I}{\partial p_k} > 0$$

These relationships are fundamental to the theory of investment: for any level of the stock of capital, the output of investment goods is positively related to the price of capital. This is the supply curve for investment goods, shown in Figure 2-2.

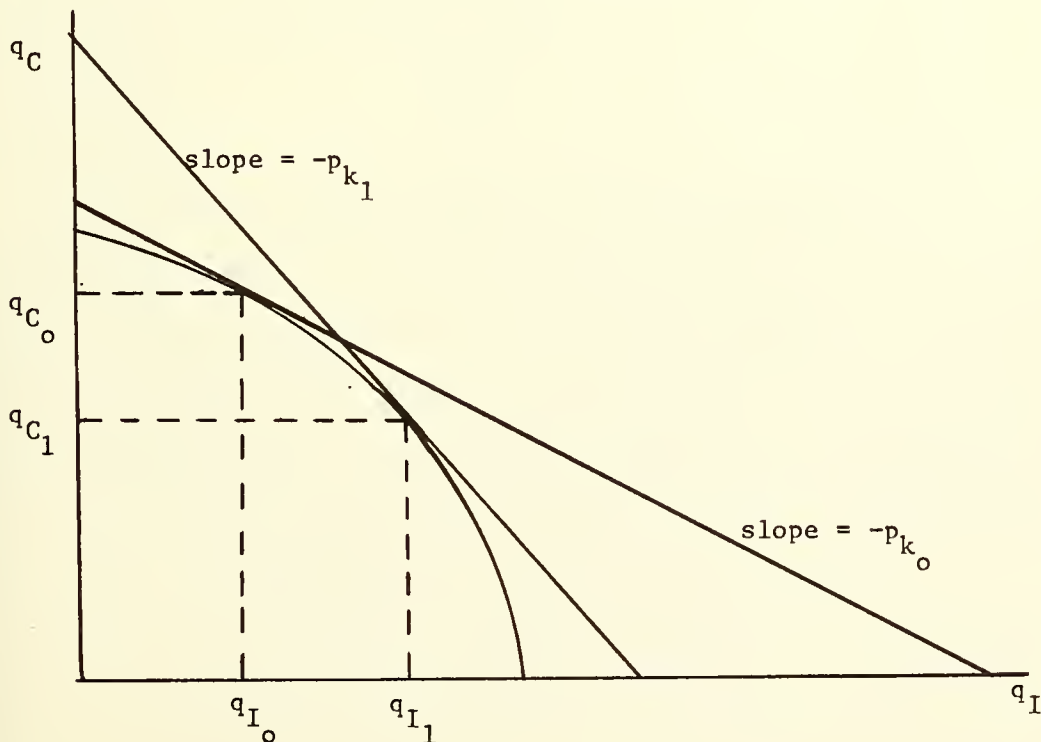


Figure 2-1

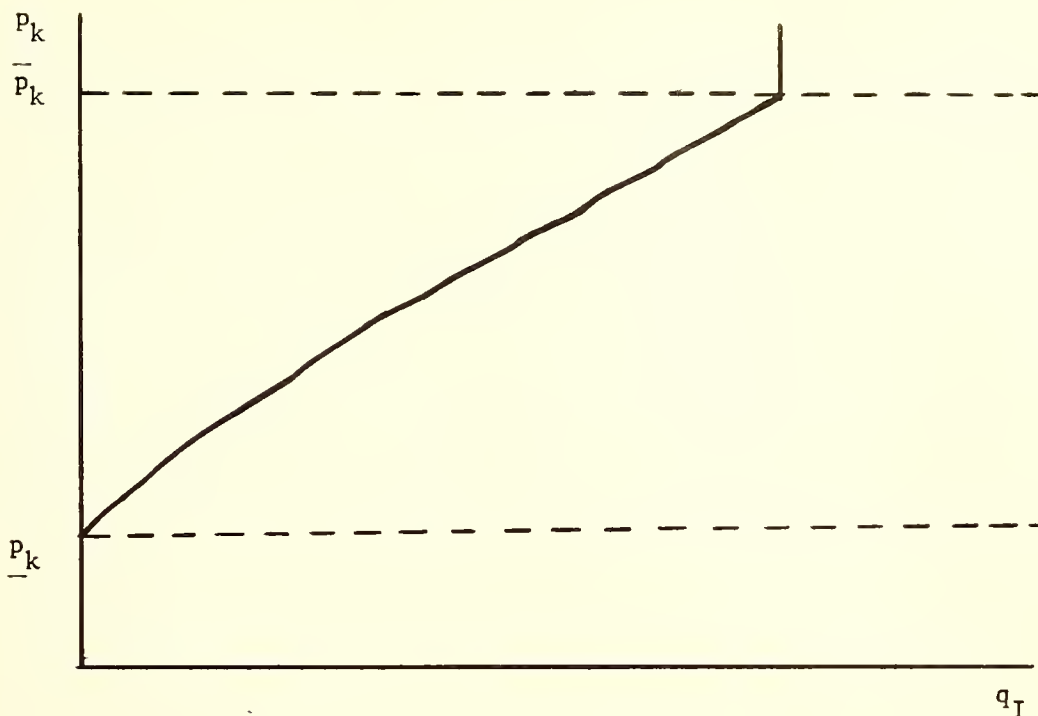


Figure 2-2

Since we assume that consumption goods are capital intensive ($k_C > k_I$), by the well-known Rybczynski theorem (1955),

$$(2.17) \quad \frac{\partial q_C}{\partial k} > 0 \quad \frac{\partial q_I}{\partial k} < 0$$

As this analysis makes clear, the market for capital services can always instantaneously come into equilibrium for any stock of capital and any price of capital. There is clearly no need here for any disequilibrium theory or any notion of the "desired" as opposed to "actual" flow of capital services. Rentals will always move so that firms are content with the existing flow of services. And nothing in the analysis depends crucially on the use of the two-sector model, or the existence of only one type of capital.

We have so far ignored the production activities of the government. We assume that the government hires the services of capital and labor in the market, paying the going market rental and wage rates, and produces a public con-

sumption good. We can think of national defense or police services as being the public consumption good produced by the government. In order not to complicate the analysis, we assume that private and public consumption goods are produced by "the same" homogeneous production functions: that is, given any relative factor prices, both commodities are produced with the same amount of capital per unit of labor.

This special assumption allows us to write the production relations for the private sector (equations (2.14) and (2.15)) as:

$$(2.18) \quad q_I = q_I(k, p_k)$$

$$(2.19) \quad q_C^P = q_C(k, p_k) - e$$

where q_C^P is the production of private consumption goods and e is the production of public consumption goods. That is, we assume that private and public consumption goods are perfect substitutes in production.

III The Demand for the Stock of Capital

The demand for capital is distinct and quite different from the demand for capital services. Capital has the dimension of a stock: so many machines or buildings. It has no time dimension. The price which clears the market for capital goods is the price of capital, and it is measured in value units per machine or building.

We note again that there is no necessary connection between ownership of capital and the use of capital services. It is possible to use capital one does not own by renting it; and possible to let out owned capital to other people to use. If we take account of this, the motive for holding capital is the stream of income which it is expected to produce in the form of rentals, implicit or actual. We want to emphasize what appear to us to be two quite different kinds of decisions: how much capital services to employ, and how much capital to own.

If we insist on income as the motive for holding capital, the demand for capital is essentially an asset demand, and the price of capital will settle so that the demand for capital as an asset equals its given supply. The fact is that very often the same firm owns and employs the same capital. When this happens, we want to divide the firm's decision into two parts, a production decision and a portfolio decision.

Portfolio decisions are an attempt by wealth owners to distribute their wealth among assets in an optimal way. The demand for assets by wealth owners in this sense is similar to the demand for consumption goods in the usual theory of consumer choice. Given the consumer's tastes, the quantity demanded of each commodity depends on the consumer's budget constraint, given by his real income, and the set of relative prices. Similarly, in the theory of asset choice, given the wealth owners' preferences, the quantity demanded or supplied of each of the assets depends on the wealth owners' budget constraint--his total wealth--and the set of asset prices and their expected rates of return.

Monetary equilibrium in the asset markets is possible only when wealth owners are just content to hold the existing stock of capital (and supplies of other assets) at going rates of return. This instantaneous equilibrium can be reached, given the value of money, by an adjustment of the price of capital and interest rates, or given the price of capital, by an adjustment of interest rates and the value of money.¹

There may actually be many combinations of the price of capital and the value of money that instantaneously equilibrate the asset market, and to each such combination, there corresponds an equilibrium interest rate. If either price is prevented from moving, the other may still be free to find an equili-

¹Cf. the earlier quotation from Keynes.

brium level. Only in the case in which both the value of money and the price of capital are prevented from changing will there be any reason to think of disequilibrium in the asset market. Only in this case is there any reason to allow for a difference between the "desired" stock of capital and the actual. If prices are flexible, either the value of money or the price of capital or both will shift to make wealth owners, or their agents the firms, content to hold the existing stock of capital at each instant.

As a specific example of this fact, consider an economy with three assets, money, bonds and capital. We can write very generally the demand functions for these as:

$$(3.1) \quad mp_m = m^d p_m = L(a, q, \rho_m, \rho_b, \rho_k); \quad 1 > \frac{\partial L}{\partial a} > 0, \quad \frac{\partial L}{\partial q} > 0, \quad \frac{\partial L}{\partial \rho_m} > 0, \quad \frac{\partial L}{\partial \rho_b} < 0, \quad \frac{\partial L}{\partial \rho_k} < 0$$

$$(3.2) \quad bp_m = b^d p_m = H(a, q, \rho_m, \rho_b, \rho_k); \quad \frac{\partial H}{\partial a} < 0, \quad \frac{\partial H}{\partial q} < 0, \quad \frac{\partial H}{\partial \rho_m} < 0, \quad \frac{\partial H}{\partial \rho_b} > 0, \quad \frac{\partial H}{\partial \rho_k} < 0$$

$$(3.3) \quad kp_k = k^d p_k = J(a, q, \rho_m, \rho_b, \rho_k); \quad 1 > \frac{\partial J}{\partial a} > 0, \quad \frac{\partial J}{\partial q} < 0, \quad \frac{\partial J}{\partial \rho_m} < 0, \quad \frac{\partial J}{\partial \rho_b} < 0, \quad \frac{\partial J}{\partial \rho_k} > 0$$

with

$$(3.4) \quad a = kp_k + (b+m)p_m = kp_k + gp_m = k^d p_k + (b^d + m^d)p_m$$

where superscript d represents "demand", m is the per-capita quantity of money, b the net quantity of government bonds outstanding, the H function is the net demand function for bonds by the private sector, p_m the price of money in terms of consumption goods (the inverse of the price level), a the per-capita value of assets, q the per-capita level of income measured in consumption goods ($q_C + p_k q_I$), subscripted ρ 's stand for the rate of return on the respective assets, and g is total government debt, including the money stock, outstanding.¹ (3.4) is the

¹In writing the per-capita demand for assets as in (3.1) to (3.3), we are ignoring "distribution effects," that is, we assume that aggregate portfolio decisions are independent of the distribution of wealth and income in the economy.

wealth constraint; at any instant the per-capita value of assets demanded must be equal to the per-capita value of assets held.

The value of assets held, a , enters the demand functions since it is the wealth constraint. We make the assumption that the marginal propensities to increase holdings of money and capital out of any increase in wealth are positive but do not exceed unity, while net holdings of bonds may either rise or fall as wealth increases.

Income enters the demand functions as a measure of the transactions demand for money. Cash balances yield a return in kind if payments and receipts do not exactly match for the average wealth holder, or if there is uncertainty about the timing of payments and a cost to switching from cash to bonds. Many measures of this return have been proposed, such as the aggregate value of transactions, or disposable income. We choose to measure it by real income in terms of consumption units, $q = q_C(p_k, k) + p_k q_I(p_k, k)$. Real income rises with the price of capital.¹

We assume that an increase in the level of income increases the demand for money. But at any given level of wealth and rates of return on assets, an increase in the demand for one asset must involve a decrease in the demand for at least one other asset. In fact we assume that neither the demand for bonds nor that for capital increases when the level of income rises.

The remaining important variables are rates of return. One dollar invested in real capital yields a return of ρ_k which has two components. The first is $r(p_k)/p_k$, the rentals rate per unit of value of capital, equal to the marginal

¹To see this, examine Fig. 2-1; income in consumption units is the intersection of the price line with the vertical axis, and this increases when p_k rises from p_{k_0} to p_{k_1} .

product of capital in the investment goods sector, which is the income wealth owners obtain from renting a dollar's worth of their capital to business firms. As we have already shown above, the rentals rate per unit of value of capital depends only on the price of capital goods; a rise in the price, by shifting resources to the labor-intensive sector, increases the capital intensity in both the consumption and the investment goods sectors and lowers $r(p_k)/p_k$.¹ The second component of the rate of return to capital is the expected capital gain on the unit of capital, equal to the rate at which the consumption price of capital is expected to increase over time, π_k . It then follows that we can write the rate of return to capital as: $\rho_k = r(p_k)/p_k + \pi_k$.¹

We assume that money holdings earn no interest. There still is, however, the possibility of capital gains and losses on money due to changes in the price level. Since we take consumption goods to be the numeraire, we work with the consumption goods price of money, p_m . This is equal to the amount of goods a single unit of money will buy, and is the inverse of p , the price level. We call the expected rate of change in p_m , π_m , and this is equal to the negative of the expected rate of inflation. An increase in the expected rate of inflation means a fall in π_m ; money will be losing value faster. The rate of return to money, ρ_m , is just equal to π_m .

To simplify the analysis we assume that bonds have a fixed demand money price and a variable interest rate, like a savings account or a call loan. Bonds may be issued by the government or by individuals and these two instruments are assumed to be perfect substitutes. We measure the per-capita quantity of bonds, b ,

¹Where there are positive transactions costs, the quantities demanded will generally depend not only on the current rental but also on the expected future path of this variable; in order to simplify the analysis, we include only the current value of this variable in the demand functions.

in money units since a bond can always be turned into one unit of money. The interest rate, i , is determined in the market so that private individuals are content to hold the net amount of bonds the government has issued. The net quantity demanded may be negative, since at low interest rates on bonds, private individuals may want to borrow from the government to hold money or capital. Since the value of a bond is fixed in money terms, changes in p_m will also give rise to real capital gains or losses on bonds. The rate of return to bonds is $\rho_b = i + \pi_m$, the interest rate plus the expected real capital gain from holding the bond.

We also assume that the three assets are gross substitutes, that is, that an increase in the rate of return on one of them raises the quantity demanded of this asset while it lowers the proportion of wealth asset holders want to invest in the other two assets.¹

It is important to note at this point that if all returns were perfectly certain, wealth owners would hold real capital and bonds only if they had the same rate of return. The two assets would be perfect substitutes and market equilibrium would require $\rho_k = \rho_b$, which in the absence of expected capital gains or losses reduces to $r(p_k)/p_k = i$, which is the Wicksellian equality of the natural and the market rate. In our model we do not assume that returns are perfectly certain, so that wealth owners, who are assumed to be risk averters, will in general diversify their portfolios.

Equilibrium in each of the three markets requires that the quantities demanded and supplied of each of the assets be equal, that is, $m = m^d$, $b = b^d$, and $k = k^d$. But from (3.4) it can be seen that if the markets for any two of the assets are in equilibrium, then the market for the third will also be in

¹The assumption of gross substitutability is made by James Tobin, "An Essay on the Principles of Debt Management" in Fiscal and Debt Management Policies, Englewood Cliffs, N.J., Prentice Hall, 1963, 143-218.

equilibrium. We can, therefore, work with any two of the markets, and we choose to exclude the bond market.

As of any instant, the expected rates of change in the prices of capital and money are given, as are the stock of capital, the stock of debt and, from the viewpoint of the private sector, the composition of the debt as between money and bonds. We call the ratio of total debt to money supply, x where

$$(3.5) \quad x = g/m$$

At any instant of time the government can change x by open-market operations, altering m while keeping g constant. We will discuss the determination of the important variables, π_k and π_m when we examine the problem of equilibrium over time.

Using (3.1) and (3.3) we can find the pairs of (p_m, p_k) that equilibrate the assets markets for any supplies of money, bonds and capital. This is the "aa" schedule of Figure 4-1, and it is in general upward sloping.¹ As the value of money, p_m , rises, total wealth rises thus increasing the demand for

¹Differentiation of (3.1) and (3.3) using numerical subscripts to indicate the partial derivatives with respect to the i th argument, yields

$$\left. \frac{dp_k}{dp_m} \right|_{aa} = \frac{\begin{vmatrix} m - L_1 g & L_4 \\ - J_1 g & J_4 \end{vmatrix}}{\begin{vmatrix} L_1 k + L_2 \frac{\partial q}{\partial p_k} + L_5 \frac{\partial \rho_k}{\partial p_k} & L_4 \\ (J_1 - 1)k + J_2 \frac{\partial q}{\partial p_k} + J_5 \frac{\partial \rho_k}{\partial p_k} & J_4 \end{vmatrix}}$$

The denominator is unambiguously negative while the numerator is definitely negative if $m > L_1 g$. Rewriting this condition as $[(L_1 a/m) g/a - 1] m < 0$, we notice that the bracketed part of the first term is the wealth elasticity of the demand for money. Since there is no empirical evidence to suggest that the wealth elasticity is even close to the inverse of g/a , which in the U.S. is around 6, we are justified in assuming that the inequality holds. Then "aa" is upward sloping.

capital; to offset this excess demand, the rate of return on capital ρ_k must fall, and so p_k must rise until wealth holders are again content to hold the existing capital stock.

We should note that a determinate interest rate corresponds to each point on aa, though we cannot in general say how the interest rate varies as we move along aa. Consider a movement up the aa schedule; the increase in p_m creates excess demand for capital and under reasonable conditions (the condition on the wealth elasticity of demand for money in the footnote above), it also creates an excess supply of money: if the increase in p_k which equilibrates the capital market still leaves excess supply in the money market, the interest rate will have to fall to equilibrate the money market (the change in the interest rate will also affect the capital market) and vice versa.

IV. The Consumption Goods Market and Instantaneous Equilibrium

For the whole system to be in equilibrium at any instant, the pair (p_m, p_k) chosen for the asset market must also equilibrate the demand and supply for consumption goods. We now come to the question of the determinants of the aggregate demand for consumption goods. A satisfactory theoretical answer to this question would describe the decision which the consumer makes in dividing his disposable income among the alternatives facing him: he can buy consumption goods or save by purchasing capital, bonds or money. At the present time this decision is well understood only on the assumption of perfect certainty about the future course of prices and rates of return. This is an assumption we do not wish to make in this paper. At this stage, we have to confess our inability to provide a satisfactory answer, and offer instead a plausible, empirically acceptable demand function without supplying any theoretical derivation of it.

We assume that consumption demand is a function of disposable income and wealth. This general formulation includes as a special case consumption functions

without wealth, so that our main lines of argument do not depend on the presence of wealth affects on consumption. We ignore variables such as rates of return which may influence consumption. Substantial rate of return influences on consumption may upset some of our findings; at the present time, though, there appears to be little empirical evidence that rate of return influences are important in comparison to the factors we do include. We write our consumption function as:¹

$$(4.1) \quad c^d = c(a, y) \quad \frac{\partial c}{\partial a} > 0, \quad \frac{\partial c}{\partial y} > 0$$

where y is disposable income.

Disposable income per capita is the sum of the following components: per-capita factor earnings, which are equal to the per-capita value of output measured in consumption goods; the value of net government taxes and transfers (including interest on bonds), which is equal to the real value of the per-capita deficit, dp_m less real government expenditure e ; and expected capital gains or losses on existing assets from changes in prices.

$$(4.2) \quad y = q_C(k, p_k) + p_k q_I(k, p_k) + (dp_m - e) + \pi_m g p_m + \pi_k k p_k$$

Thus for the consumption goods market to clear, we have

$$(4.3) \quad c^s = q_C - e = c(a, y) = c\{a, [q_C + p_k q_I + (d + \pi_m g)p_m + \pi_k k p_k - e]\} = c^d$$

From (4.3) we obtain a "cc" schedule of pairs (p_m, p_k) which clear the consumption goods market. An increase in p_k will produce excess demand for consumption goods on three counts: first, it reduces production of these goods; second, it increases the value of wealth and the wealth effect leads to an increase in

¹ We thus assume that private and public consumption goods are independent goods in consumption; our earlier assumption was that they were perfect substitutes in production.

the demand for consumption goods; and third, it increases income measured in consumption goods. On the other hand, if π_k is negative, an increase in p_k would reduce demand by reducing the value of expected capital gains, or by increasing the amount of capital which has to be devoted to maintaining the stock of wealth. We shall assume that the first three factors outweigh this fourth effect if π_k is negative.¹ Of course, if π_k is positive, there is no ambiguity. An increase in p_m tends to increase consumption demand by increasing the value of wealth, and also tends to increase demand if $(d + \pi_m g)$ is positive.² Thus if $(d + \pi_m g)$ is positive, an increase in p_m definitely produces excess demand in the consumption goods market. In this case, an increase in p_k would have to be accompanied by a fall in p_m to maintain equilibrium in the consumption goods market. The resultant cc schedule is shown in Figure 4-1.

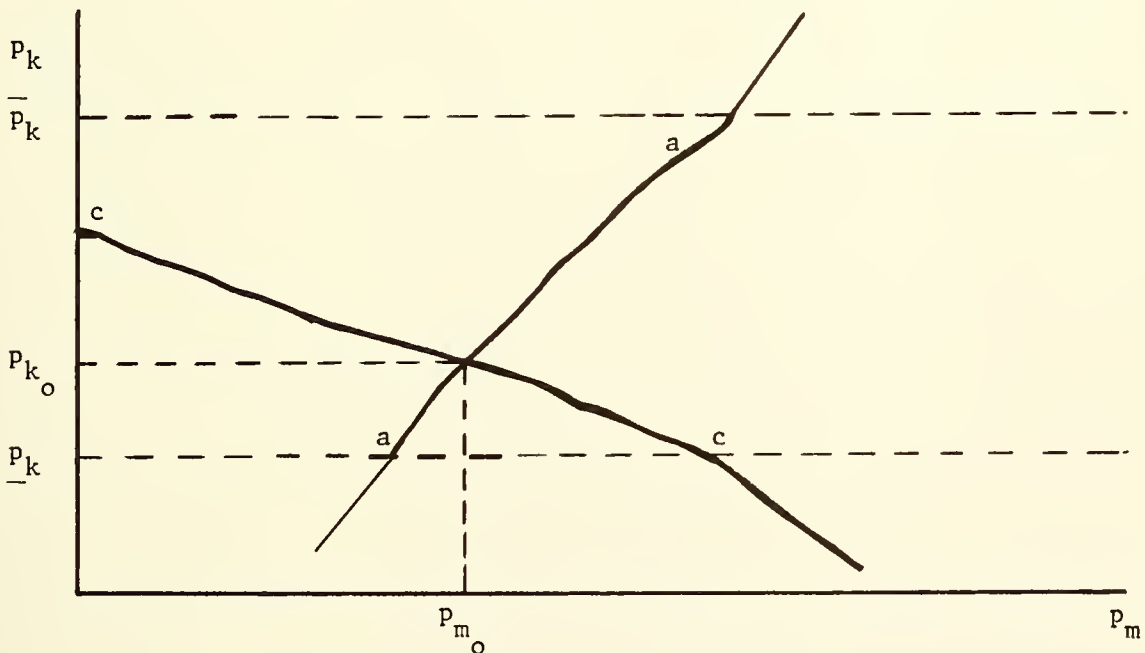


Figure 4-1

¹In fact, in the steady state p_k will be constant so that we would expect π_k to be equal to zero. Then the possible ambiguity arising from the π_k term disappears.

²The effects of an increase in p_m are ambiguous in the case where $(d + \pi_m g)$ is negative. We do not analyse this case here since the typical long-run p_m behavior of an economy with a positive outstanding stock of government debt requires that $(d + \pi_m g)$ be positive to maintain a positive real debt over time.

The intersection of the aa and cc schedules gives the prices of capital and money, p_{k_o} and p_{m_o} , which equilibrate the assets and consumption goods markets. A determinate interest rate corresponds to the pair (p_{k_o}, p_{m_o}) so that the instantaneous equilibrium of the economy is fully determined. The equilibrium price of capital also determines the flow supply of investment goods. Every market is in equilibrium, there is no divergence of a "desired" stock of capital from the existing one, but profit maximizing suppliers of investment goods will generally be producing a flow of new investment. The higher the equilibrium price of capital the larger this supply will be.

V. Government Policy

The government can use its policy variables to influence the positions of the aa and cc schedules and determine the equilibrium prices of capital and money and the output of investment goods. Monetary policy operates through open market operations, changing the composition of the outstanding government debt. An open market purchase, for instance, increases the supply of money and reduces that of bonds. A pure fiscal policy would be that of the "marginally balanced budget" in which the deficit is kept constant and net taxes and expenditures are varied; we call this "pure" because it leads to no changes in the supply of debt over time compared with what that stock would otherwise have been. A deficit financed fiscal policy is one which involves changes in the deficit and so over time produces a different debt from that which would otherwise have prevailed.

First we consider the effects of an open market purchase. This shifts the aa schedule upward in Fig. 5-1 from $a_o a_o$ to $a_1 a_1$, because such a purchase causes an excess supply of money which can be offset by a higher price of capital at any given price of money. This upward shift in aa results in a higher p_k , thus leading to an increase in the output of investment goods. It is also inflationary in that p_m , the inverse of the price level, falls.

The only immediate effects of fiscal policy are on the position of the cc schedule. Either a balanced budget fiscal policy which raises both government expenditure and taxes, while keeping the deficit constant, or a deficit financed policy which increases d , will produce excess demand in the consumption goods market. At any given level of p_m this can be offset by a fall in p_k . Thus either of these types of fiscal policy will shift the cc schedule downward from c_0c_0 to c_1c_1 . The result is a lower p_k and a lower p_m . The output of investment goods will fall, but there is no certainty that the interest rate will rise as conventional accounts lead us to expect. As the position of cc shifts, the economy moves along the aa schedule, and we showed above that there is no presumption about the way the interest rate changes along aa. Even ex-post, there may be no consistent relationship between the level of investment and the interest rate.

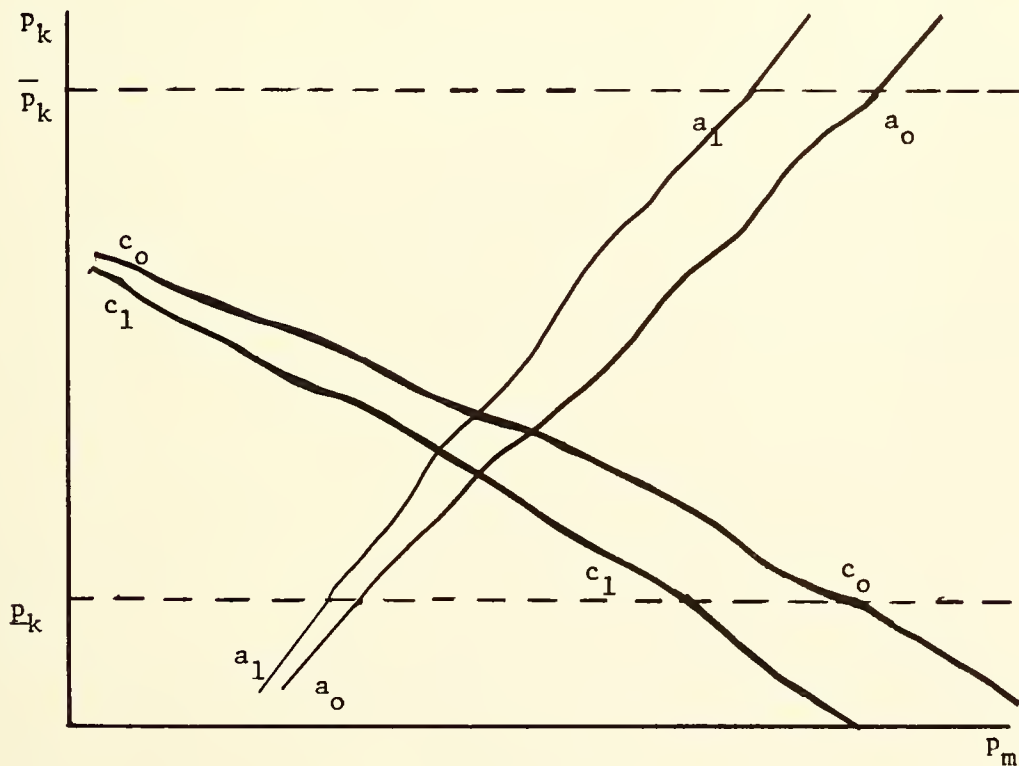


Figure 5-1

VI Stock-flow Equilibria

Once the producers' equilibrium level of output in the investment goods sector is determined, it is natural to ask whether wealth owners will absorb this real capital into their portfolios at the equilibrium prices and interest rate. We can ask the same question about the government deficit, which increases the outstanding stock of government debt. The fact that actual and desired stocks of capital, money and bonds are equal at certain prices does not guarantee that people will be content to absorb any given increases in the stocks, even though the demand and supply of consumption goods are equal. If people are not content to absorb the given additions, how will the new supplies of capital and other assets find room in private portfolios?

To begin with we note that the private sector income budget constraint requires that private disposable income equal the private demand for consumption goods plus the value of desired additions to asset holdings.

$$(6.1) \quad q + vp_m = c^d + \frac{\dot{K}^d}{N} p_k + \frac{\dot{G}^d}{N} p_m$$

where $G = Ng$ and v is the per-capita nominal net transfers to the private sector including interest on government bonds.

The budget deficit is in turn equal to net transfers plus government expenditures,

$$(6.2) \quad dp_m = vp_m + e$$

and we know that

$$(6.3) \quad q = q_I p_k + q_C = \frac{\dot{K}^s}{N} p_k + q_C$$

Using these two facts in (6.1) we get

$$(6.4) \quad \frac{\dot{K}^s}{N} p_k + q_C - e + dp_m = c^d + \frac{\dot{G}^d}{N} p_m + \frac{\dot{K}^d}{N} p_k$$

Since the deficit is equal to the rate of increase in the government debt, we can write (6.4) as

$$(6.5) \quad \frac{\dot{K}^s}{N} p_k + \frac{\dot{G}^s}{N} p_m + q_C - e = c^d + \frac{\dot{K}^d}{N} p_k + \frac{\dot{G}^d}{N} p_m$$

But when the consumption market clears, $q_C - e$ is equal to c^d , so that we get the equality of total supply of new assets and total desired acquisitions.

$$(6.6) \quad \frac{\dot{K}^s}{N} p_k + \frac{\dot{G}^s}{N} p_m = \frac{\dot{K}^d}{N} p_k + \frac{\dot{G}^d}{N} p_m$$

If we add the capital gains terms to both sides of (6.1), we see that consumption market equilibrium implies that at the equilibrium prices desired and actual saving are equal. Rewriting (6.6) we have

$$(6.7) \quad p_k [q_I - \frac{\dot{K}^d}{N}] = p_m [\frac{\dot{G}^d}{N} - d] = p_m [\frac{\dot{G}^d}{N} - \frac{\dot{G}^s}{N}]$$

This indicates that it is possible for individuals to be content to hold existing stocks of capital and debt, to purchase the quantities of consumption goods desired, and therefore to be accumulating in total the value of assets they wish to accumulate, but at the current equilibrium prices to desire to add to their stocks in proportions which are different from the rates at which these stocks are being supplied. From (6.7) we see that if the government is increasing the supply of debt more rapidly than wealth holders wish to accumulate it at the current prices, wealth holders will be accumulating capital more slowly than they wish to: a flow excess supply of debt is accompanied by a flow excess demand for capital.

In fact even if $\frac{\dot{G}^d}{N} = d$, so that wealth holders are accumulating just the amounts of capital and debt that they desire, they may not be accumulating debt in the desired proportions. We have

$$\frac{\dot{G}^d}{N} = \frac{\dot{M}^d}{N} + \frac{\dot{B}^d}{N}$$

and

$$d = \frac{\dot{G}^s}{N} = \frac{\dot{M}^s}{N} + \frac{\dot{B}^s}{N}$$

and the equality of $\frac{\dot{G}^d}{N}$ and d does not imply that the actual rate of change of the quantities of money and bonds are equal to desired rates of change.

The results of any difference between actual and desired rates of change of asset holdings will clearly be changes in the price of capital (and its rate of return), the price of money, and the interest rate over time. If we insist on a complete dynamic model in which the asset markets are always in equilibrium, the supply of capital at every instant is the integral of past investment, and the supply of debt is the integral of past deficit, then the three variables p_k, p_m and i must follow paths which allow for the voluntary absorption of new capital and new debt. Prices may be changing, but at every instant all markets are in equilibrium and there is a determinate rate of investment.

The growth of portfolios and the absorption of capital and debt by saving are dynamic processes which can be studied only through time and which generally involve changes in the prices of capital and money and the interest rate. Accordingly, we must now proceed to a full dynamic analysis of this economy.

VII. Equilibrium through Time and Expectations

We have proposed a theory of the determination of the instantaneous rate of investment which separates firm decisions into three parts: a producer's decision as to the amount of capital to employ, a portfolio decision as to the amount of capital to own, and a supply decision as to the rate of production of capital goods. This thorough-going equilibrium analysis has brought us to the paradox that the flow supplies and demands of assets may not be equal at the current equilibrium prices.

Before we discuss the solution of this paradox, one example may be helpful: the housing market. At any instant of time the services of the housing stock

are offered inelastically and the rental to housing is determined by the demand for these services; at the same time the existing stock of houses must be held in wealth owners' portfolios and the price for houses will be that price which just makes wealth owners content to hold the existing stock. Builders make their decision to supply new houses on the basis of the going market price. As these houses come onto the market, they will be absorbed into portfolios but if, at the existing price, the rate of change of the stock demand is not equal to the flow supply, the price of houses will have to change over time to accommodate these increases in the stock.

The moral of this example is that the price of capital and/or the value of money will have to change through time as we move through a succession of instantaneous equilibria. Let us focus for a moment on the idea of a succession of instantaneous equilibria. These will form a path for all the variables in the system. Is any path which satisfies the instantaneous equilibria at every moment admissible? Clearly not. There are important restrictions on the way the instantaneous equilibria fit together. In a continuous time model these are restrictions on the derivatives of the variables.

For example, the capital stock at any instant is the integral of past instantaneously determined rates of investment. Likewise, the stock of outstanding debt is the integral of past deficits. These restrictions turn the instantaneous equilibria into a system of differential equations.

At any instant the rate of change of the per-capita stock of capital is equal to the per-capita output of investment goods minus the amount of investment needed to provide individuals entering the economy with the existing per-capita level of capital.

$$(7.1) \quad \dot{k} = q_I(k, p_k) - nk$$

where n is the rate of increase of the labor force.

Similarly, the rate of change of per-capita government debt is equal to the deficit minus the amount of new debt needed to maintain a constant amount of debt per capita

$$(7.2) \quad \dot{g} = d - ng$$

Together with (3.1), (3.3), (3.5) and (4.3), these constitute a system of six equations in eleven unknowns, k , g , m , x , p_k , i , p_m , π_k , π_m , d and e . Obviously such a system has many possible solutions; it is underdetermined. Any paths for the eleven variables which satisfy the six equations are admissible. On each such path p_k and p_m will be changing so that equilibrium in the assets and commodity markets is always achieved. The paradox of the last section is partially resolved. There is no need for desired and actual flows of assets to be equal at current equilibrium prices if asset prices are free to move.

But they are not free to move arbitrarily because the actual rates of change in p_m and p_k will influence the expected rates π_m and π_k . There are three popular models of this process, "static expectations," "adaptive expectations" and "perfect foresight."

"Perfect foresight" requires $\pi_m = \frac{\dot{p}_m}{p_m}$ and $\pi_k = \frac{\dot{p}_k}{p_k}$ at each moment on the whole path. If in addition we were to specify equations describing the time paths of the policy variables d , x and e , we would have a complete system of eleven equations in eleven unknowns, determining fully the time path of the economy.

While the assumption of perfect foresight in problems of intertemporal economics seems to be a natural extension of the assumption of perfect information usually made in static equilibrium models, it imposes severe restrictions on the system and its path. First, it rules out all notion of uncertainty and portfolio diversification; second, it requires assumptions about information

which are unlikely to be met in reality; and third, and most important, it leaves almost no room for discretionary government policy. A strong condition on government policy which is consistent with perfect foresight is that future government policy is known and therefore unalterable. In this case, the complete paths for policy variables are determined from time zero, and no further change can occur without violating the perfect foresight assumption.

Government discretionary policies are consistent with perfect foresight when they do not induce regrets in economic agents. But even with this weaker condition, most of the policies considered in the analysis of Section V, which seem to be the types of policy most government use to control aggregate demand and its composition, would be excluded because they do induce regrets.

In the other models of expectation formation, where actual price changes influence expected price changes either not at all or with a lag, there will not be an intertemporal competitive equilibrium, only a succession of instantaneous equilibria based on possibly wrong guesses about the future. There is a larger sphere for discretionary government policy and the set of solution paths becomes wider. There will also be regrets which imply that individuals and firms are not in intertemporal equilibrium. They will be in instantaneous equilibrium given their imperfect information about the future.

It seems to us, then, that the requirement of intertemporal competitive equilibrium is very strong, and requires assumptions about information which are unlikely to be met in reality. There seems to be a place for a theory that allows for a lack of intertemporal equilibrium while insisting on instantaneous equilibrium.

Static expectations models, however, are too naive, particularly where prices may actually be changing in the economy. Accordingly, in the following section we analyze two models using the adaptive expectations hypothesis which allows for errors and attempts to correct these errors on the basis of newly available information.

VIII. Dynamics

In order to analyze the relationship between the prices of capital and money, the rate of interest and stocks of different assets over time, we consider two simple dynamic models. In both of them we assume that the government actively manipulates either the composition of the debt (monetary policy) or the levels of expenditure and taxes (marginally balanced budget fiscal policy) to achieve a stable consumer price level, so that

$$(8.1) \quad p_m = p_m^*$$

To simplify matters, we assume that the government maintains constant the outstanding stock of nominal debt by fixing its deficit at the appropriate level.

$$(8.2) \quad d = ng^*$$

Given a constant price level over time, it is reasonable to assume that the expected rate of change of p_m is equal to zero:

$$(8.3) \quad \pi_m = 0$$

If monetary policy is used to stabilize the price level and fiscal policy is passive we have an additional equation

$$(8.4a) \quad e = e^*$$

while if fiscal policy is used to control the price level and monetary policy is passive, the additional equation is

$$(8.4b) \quad x = x^*$$

We have two possible models, depending on whether we use (8.4a) or (8.4b); both of them are summarized in systems of ten equations (the six mentioned in section VII plus (8.1) through (8.4)) in the eleven unknowns.

In both models we are missing an equation describing the process by which wealth owners and consumers form their expectations about the rate of change in the price of capital. Since they are not assumed to have perfect foresight, they will tend to make mistakes which they will probably try to correct as new information becomes available to them. A simple model of this type is the adaptive expectations model in which the rate at which people adjust their beliefs about the rate of change of the price of capital depends on the error made in predicting the current rate of change:

$$(8.5) \quad \dot{\pi}_k = \beta \left[\frac{\dot{p}_k}{p_k} - \pi_k \right]$$

A standard policy argument is that an easy monetary policy combined with a tight fiscal policy will promote growth, while a tight monetary policy together with easy fiscal policy encourages consumption at the expense of investment. The degree of ease or tightness is probably thought of in terms of the level of "the" interest rate or the general level of interest rates. We have shown, however, that there is no necessary relationship between the interest rate and the price of capital which determines the output of investment goods at each instant. We choose to define an easy monetary policy in terms of the composition of the debt: an increase in the proportion of money in the stock of outstanding debt--a fall in x --represents an easing of monetary policy. A tightening of fiscal policy is represented by a decrease in the level of government expenditures and taxes while the deficit is kept constant or by a fall in the deficit.

In this section we shall consider the policy argument outlined above: first, we examine the effects of a decrease in the debt-money ratio where a marginally balanced budget fiscal policy is used to stabilize the price level; and second, we consider the effects of an increase in government expenditures when monetary policy is used to stabilize the price level.

A. Stabilization through a Marginally Balanced Budget

In this case, given the stock of capital inherited from the past and given the expected rate of capital gains based on the past behavior of the price of capital, equations (8.1) and (8.2) determine the price of capital and the interest rate which equilibrate the assets markets.

$$(8.6) \quad p_k = \Psi(k, \pi_k; g^*p_m^*, x_o) \quad \text{with} \quad \frac{\partial p_k}{\partial k} < 0, \quad \frac{\partial p_k}{\partial \pi_k} > 0, \quad \frac{\partial p_k}{\partial x} < 0 \quad (1)$$

Given p_k determined in the assets markets, equation (4.3) indicates the level of government expenditure, e , consistent with equilibrium in the commodity market for p_m^* and the asset market equilibrium price of capital.

Differentiating equation (8.6) with respect to time and substituting into (8.5) we have

$$(8.7) \quad \dot{\pi}_k = \frac{\beta \left[\frac{1}{p_k} \frac{\partial \Psi}{\partial k} \dot{k} - \dot{\pi}_k \right]}{\left[1 - \beta \frac{\partial \Psi}{\partial \pi_k} \frac{1}{p_k} \right]}$$

Substituting (7.1) into (8.7) we can rewrite the basic differential equations of the model as

$$(8.8) \quad \dot{k} = q_I[k, \Psi(k, \pi_k; g^*p_m^*, x_o)] - nk$$

$$(8.9) \quad \dot{\pi}_k = \frac{\beta \left\{ \left(\frac{\partial \Psi}{\partial k} \frac{1}{p_k} \right) (q_I[k, \Psi(k, \pi_k; g^*p_m^*, x_o)] - nk) - \dot{\pi}_k \right\}}{\left[1 - \beta \frac{\partial \Psi}{\partial \pi_k} \frac{1}{p_k} \right]}$$

The $\dot{k} = 0$ and $\dot{\pi}_k = 0$ lines in Figure 8-1 indicate the pairs (k, π_k) that make \dot{k} and $\dot{\pi}_k$ respectively equal to zero. Figure 8-1 corresponds to the case in which the denominator in equation (8.9) is positive; that is, the case in which the lag in the adjustment of expectations is sufficiently large to avoid

¹The signs of these derivative may be confirmed by differentiation of (3.1) and (3.3).

the perpetuation of runaway boom in the assets markets.¹

It is important to note that in this model, the more slowly that wealth owners adjust their mistaken expectations, the more likely it is that the long-run balanced growth path is stable.^{2,3}

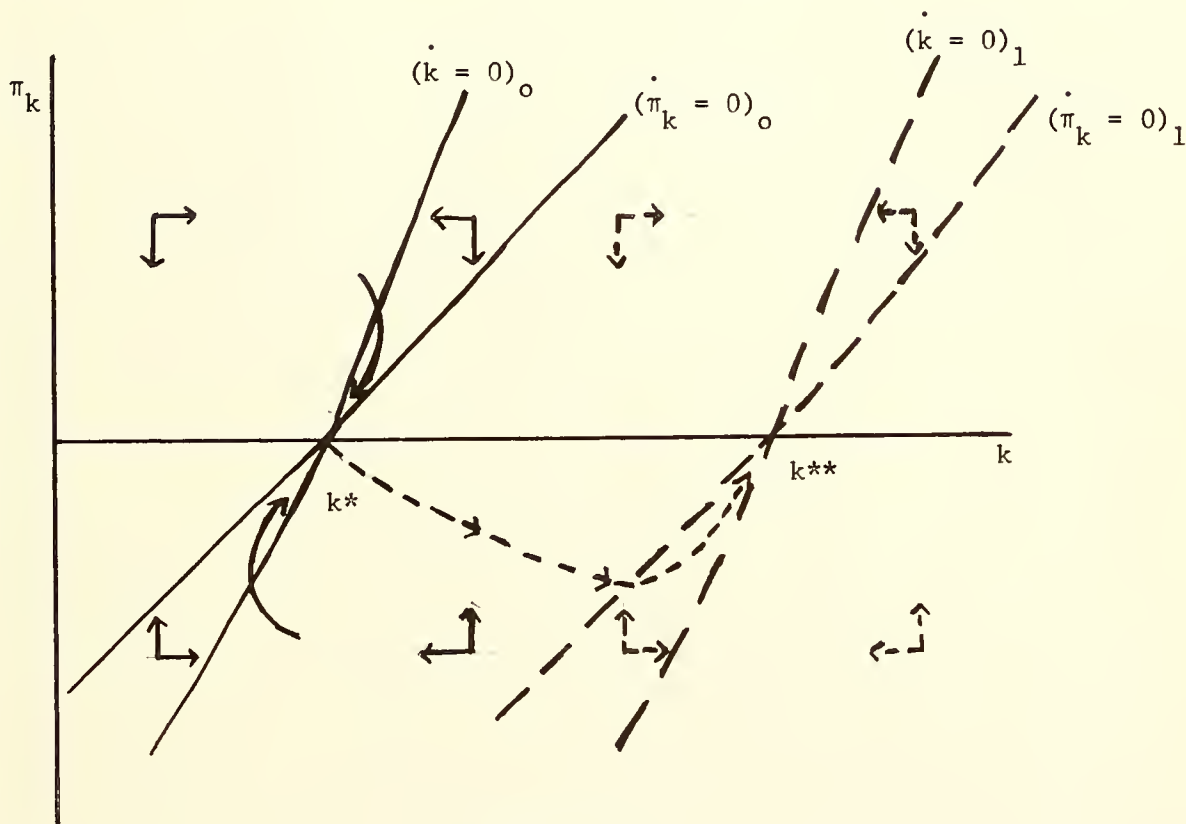


Figure 8-1

¹ A similar stability condition is also to be found in Philip Cagan, "The Monetary Dynamics of Hyperinflation" in Studies in the Quantity Theory of Money, Milton Friedman, ed., Chicago, University of Chicago Press, 1956; and Sidrauski, op.cit.

² It can be proved that in this case under rather weak assumptions the equilibrium exists and if it exists it is unique.

³ Problems of stability in the assets markets arising from a rapid adjustment of expectations lie behind the recent controversy on models with heterogeneous capital goods. See F.H. Hahn, "Equilibrium Dynamics with Heterogeneous Capital Goods," Quarterly Journal of Economics, Vol. LXXX, No. 4 (Nov. 1966), 633-646, and Karl Shell and Joseph E. Stiglitz, "The Allocation of Investment in a Dynamic Economy," Quarterly Journal of Economics, Vol. LXXXI, No. 4 (Nov. 1967), 592-609.

We may now use the model to consider the effects of an easing of monetary policy: that is, an increase in the proportion of money in the debt, equivalent to a fall in x . We examine only the stable case shown in Figure 8-1. An open market purchase which produces a fall in x leads to an increase in the price of capital which clears the assets markets. In terms of the diagrams, this shifts both the $(\dot{k} = 0)$ and $(\dot{\pi}_k = 0)$ schedules to the right since the output of investment goods will now be higher as of any pair (k, π_k) and a higher capital stock will be needed to absorb the additional output of investment goods in the steady state. The capital stock increases continually to its new higher level so that the overall rate of growth will be higher in the period of disequilibrium than in the steady state. The expected rate of increase in the price of capital initially falls below zero as the accumulation of capital forces p_k down after its first upward jump. The fall in π_k depresses p_k even further for awhile, but the depressing effect on p_k of the rise in k diminishes as k approaches its steady state value and \dot{k} approaches zero. In the end π_k , following the actual rate of change in p_k , moves back to zero.

What is required of fiscal policy in order to stabilize the price level following the rise in the price of capital and the subsequent accumulation of capital? For the reasons outlined in Section 4, an increase in the price of capital has an inflationary effect in the consumption goods market. To offset the effects of the initial rise in the price of capital, then, fiscal policy has to be tightened--the level of government expenditures and taxes has to be reduced. This is what the conventional accounts lead us to expect. Then as capital accumulates over time, the price of capital in the assets markets begins to fall from its new level so that fiscal policy can be eased on this account. However, the accumulation of capital also affects the equilibrium of the consumption goods market though the effects are ambiguous since an increase in the capital stock increases supply and increases demand through wealth and income effects. After the initial

tightening the direction of fiscal policy is uncertain.

B. Stabilization through Monetary Policy

In this case given the stocks of government debt and capital inherited from the past, π_k and p_m^* and the government's policy parameters e and d , the price of capital is determined in the consumption goods market (4.3).

$$(8.10) \quad p_k = \hat{\phi}(k, \pi_k; g^*p_m^*, dp_m^*, e), \quad \frac{\partial \phi}{\partial k} > 0, \quad \frac{\partial \phi}{\partial \pi_k} < 0, \quad \frac{\partial \phi}{\partial e} < 0$$

Given p_k determined in the consumption goods market, the government has to vary the composition of the debt in such a way as to ensure that the p_k determined by (8.10) together with p_m^* clear the assets markets.

Differentiating (8.10) now with respect to time and substituting into (8.5) and using (7.1), we obtain

$$(8.11) \quad \dot{k} = q_I[k, \phi(k, \pi_k; g^*p_m^*, dp_m^*, e)] - nk$$

$$(8.12) \quad \dot{\pi}_k = \frac{\beta \left\{ \left(\frac{\partial \phi}{\partial k} \cdot \frac{1}{p_k} \right) (q_I[k, \phi(k, \pi_k; g^*p_m^*, dp_m^*, e)] - nk) - \pi_k \right\}}{\left[1 - \beta \frac{\partial \phi}{\partial \pi_k} \frac{1}{p_k} \right]}$$

There are now three possible stable configurations of the $(\dot{k} = 0)$ and $(\dot{\pi}_k = 0)$ loci depending first, on whether an increase in the capital stock increases or decreases the equilibrium price of capital in the consumption goods market; and second, on whether an increase in the expected rate of change of the price of capital increases or decreases the rate at which that expected rate is changing (i.e., whether or not $\frac{\partial \dot{\pi}_k}{\partial \pi_k} > 0$). It is a necessary condition for stability that an increase in the capital stock decrease the rate of change of the capital stock (i.e. $\partial \dot{k} / \partial k < 0$). This condition need not always be met in practice since an increase in the capital stock may increase the equilibrium price of capital in the consumption goods market and in this way increase the output of investment goods. There are two factors working in the opposite

direction. First, the increase in k by itself lowers q_I . Second, the increase in k raises \dot{k} and lowers $\dot{\pi}_k$. If an increase in k produces so great a rise in $\dot{\pi}_k$ through the consumption market that it overwhelms these negative factors, the system will be unstable.

We show the three possible stable equilibria in Figures 8-2a, 8-2b and 8-2c. In each diagram the $(\dot{k} = 0)$ locus is downward sloping since an increase in the capital stock reduces the rate of change of the capital stock, so that a decrease in π_k --which increases the price of capital at which the consumption market clears--is required to offset this effect. The horizontal arrows indicate that $\frac{\partial \dot{k}}{\partial k} < 0$.

While the $\dot{k} = 0$ schedule must slope downward near a stable equilibrium, there are two possibilities for the $\dot{\pi}_k = 0$ schedule. First, there is the case where $\frac{\partial \dot{\pi}_k}{\partial \pi_k} < 0$. In this case the $\dot{\pi}_k = 0$ schedule must have a higher slope than the $\dot{k} = 0$ schedule near a stable equilibrium, as illustrated in Figures 8-2a and 8-2b. There are no oscillations possible in the former case. Second, is the case where $\frac{\partial \dot{\pi}_k}{\partial \pi_k} > 0$. The stable equilibria on this assumption are like the one shown in Figure 8-2c, and the system can produce cycles.

We are now ready to examine the effects of a tightening of fiscal policy, that is, a decrease in the level of government expenditures and taxes. A decrease in the level of government expenditures with a constant deficit is deflationary in the consumption goods market so that the price of capital which clears that market rises as of any pair (k, π_k) . Around a position of stable equilibrium this will shift the $(\dot{k} = 0)$ and $(\dot{\pi}_k = 0)$ schedules to the right, increasing the equilibrium stock of capital since a higher capital stock is now needed to absorb the higher output of investment goods in the steady state. If the economy were initially at a position of stable long-run equilibrium, the capital stock increases to its new equilibrium level. As can be seen from the diagrams, the movement to the new equilibrium capital stock may involve cycles.

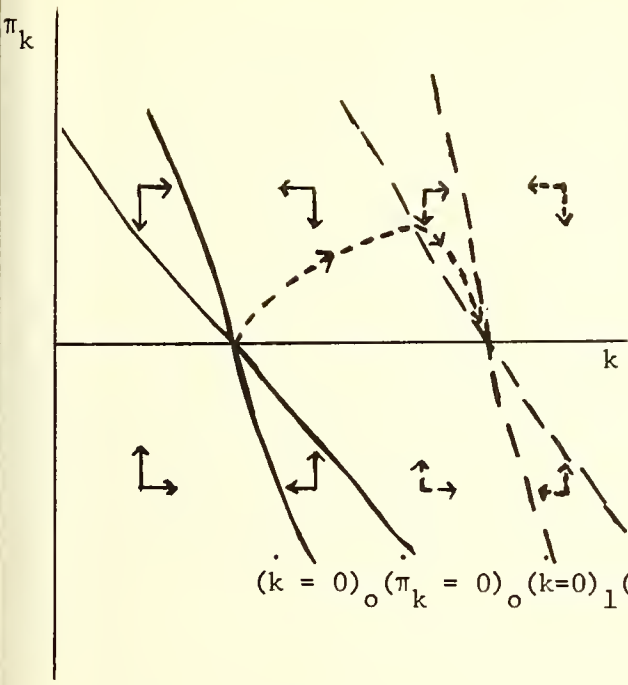


Figure 8-2a

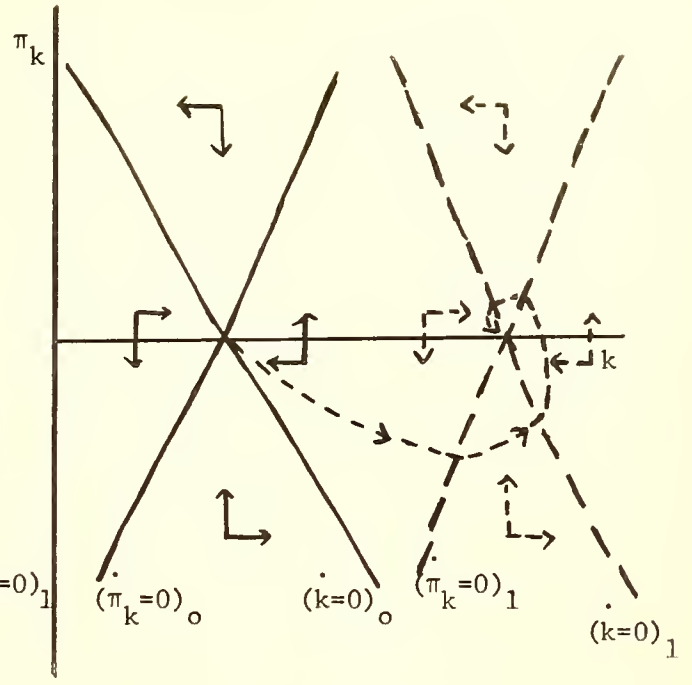


Figure 8-2b

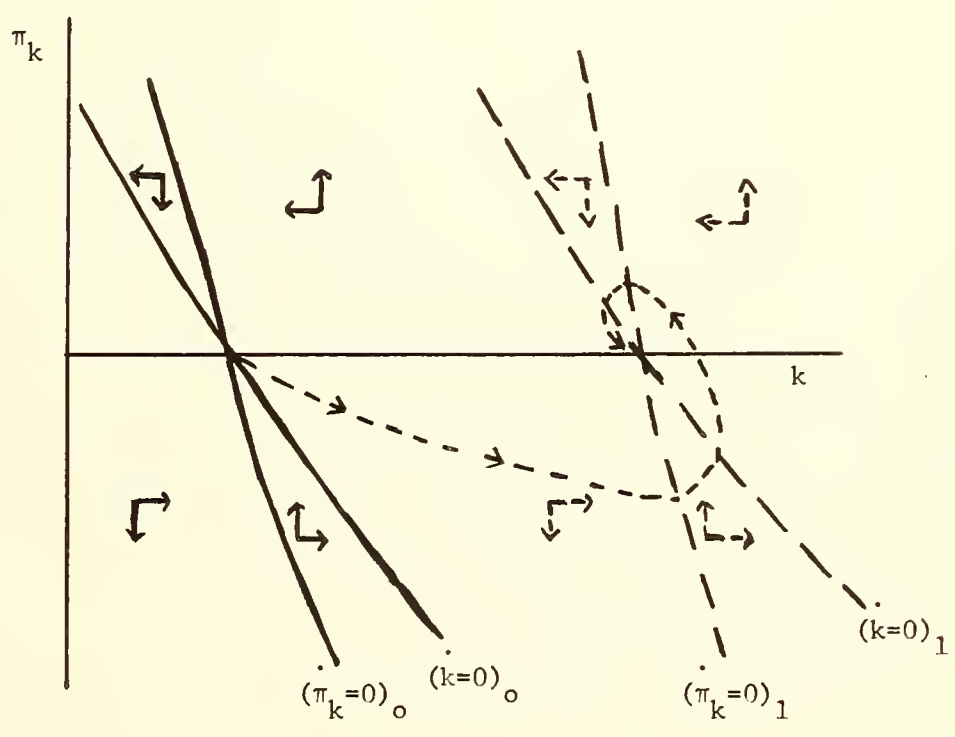


Figure 8-2c

The long-run effect of the tighter fiscal policy is a higher stock of capital, just as the long-run effect of easier money in our previous example was a higher stock of capital. At first the accompanying monetary policy must be easy to prevent changes in the price level, but the continuing changes necessary in monetary policy depend on the effect of the growing capital stock on the consumption market. If as the capital stock increases it raises the consumption market equilibrium p_k , monetary policy must always be getting easier, to achieve the necessary p_k with the larger capital stock. If a rise in the capital stock lowers the consumption market equilibrium p_k , x may have to move in different directions at different times.

These two experiments partly confirm the policy argument with which we began this section. In the short run tightening fiscal policy and easing monetary policy will raise the rate of growth. In the long run, however, one of these policies may have to be reversed as capital accumulates to maintain a stable price level.

IX. Conclusion

In this paper we have attempted to tie together an equilibrium theory of investment, Keynesian stabilization policies and the neo-classical two-sector model of economic growth to get a coherent view of the behavior of modern indirectly controlled economics. The particular models of government behavior and expectations formation we used in the last section do not exhaust the potential of models based on these ideas. We encourage the reader to "roll his own" model out of the fixings we have prepared.

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